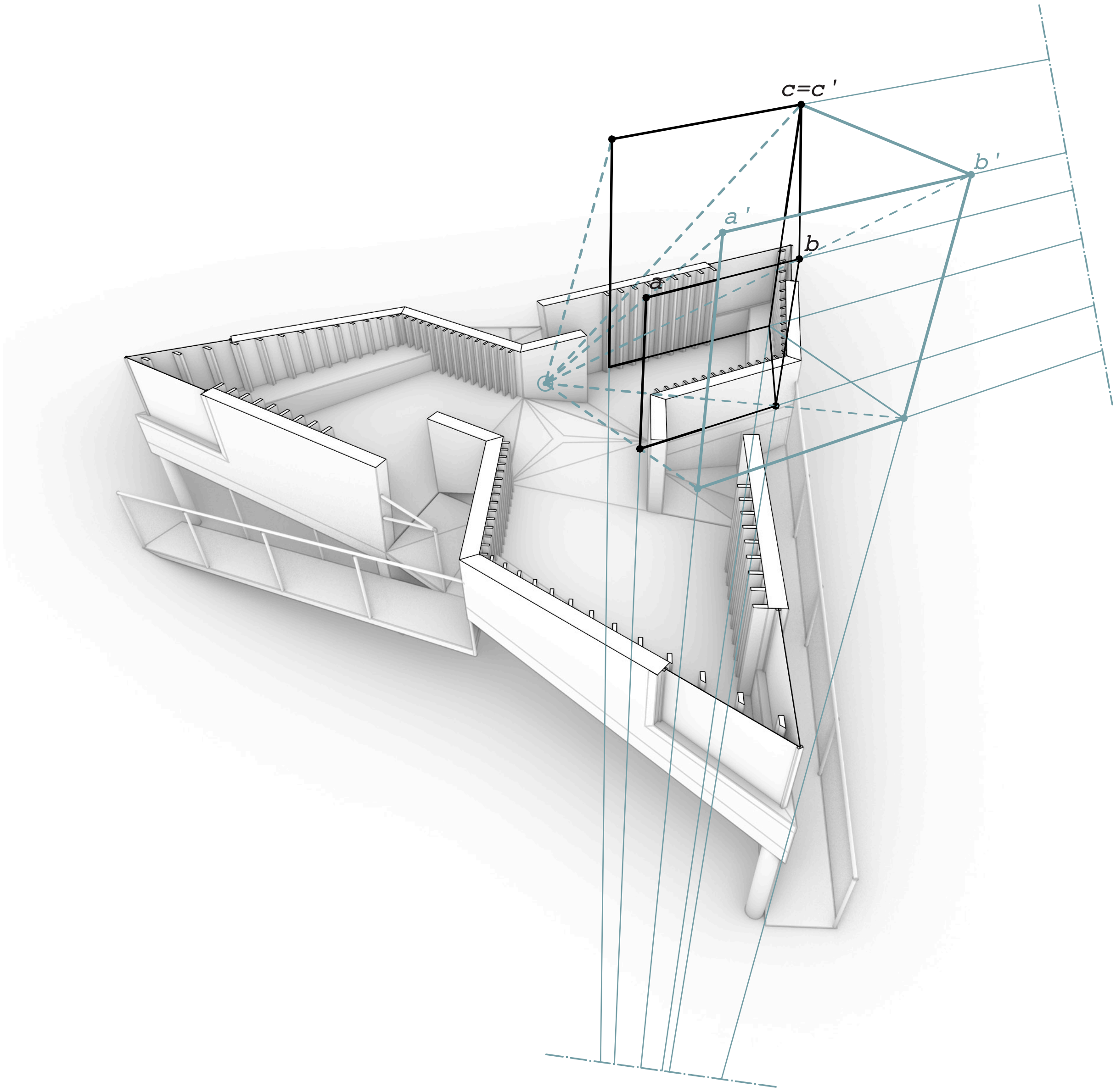
Deceptive Geometries

Spatial analysis and design through the study of visual illusions



Daniel Jonsson

Examiner: Daniel Norell

Supervisor: Malgorzata Zboinska

Chalmers School of Architecture

Department of Architecture and Civil Engineering

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Deceptive Geometries - Spatial analysis and design through the study of visual illusions

Master's Thesis 2022

Chalmers School of Architecture
Department of Architecture and Civil Engineering
Architecture and Urban Design; MPARC
Gothenburg, Sweden

Author: Daniel Jonsson
Examiner: Daniel Norell
Supervisor: Malgorzata Zboinska

"Let no one ignorant of geometry enter here!"
- Plato

ABSTRACT

Not everything is as it seems. Our perception of our surroundings, and consequently our understanding of reality, is merely a product of our own interpretation. Our eyes only receive two-dimensional projections of the three-dimensional world, which our brains then combine and reinterpret as something three-dimensional. Therefore, our perception can also easily be deceived. This has been used by architects throughout history, either to create amazement or to enhance certain architectonic features.

Deceptive Geometries is an umbrella term, coined by the author, for all geometries that in some way feel unintuitive or deceive your perception of reality. This thesis aims to define by example what such geometries can be, and investigate how they can be integrated in architectural design to choreograph spatial experiences.

The conventional way of representing a building with orthographic projections, i.e. plan and façade drawings, is convenient for describing the geometry in a compact manner. However, it does not reflect how the building is perceived in reality. For architecture in general, and geometric visual illusions in particular, the relation between subject and object plays a central role to the experience, as the vantage point of the observer is crucial to the perception of space. Thus, an extension to descriptive geometry is developed, with the purpose to capture the *perceived* geometry in a progression through space, and to decipher visual illusions.

The thesis consists of an exploration of deceptive geometries and related phenomena, followed by the development of a representation method. The two parts are then synthesized into a speculative design proposal for a building located by the Three-Country Cairn, where the borders of Sweden, Norway and Finland meet. Here, a symbolic representation of the tripoint is created, serving as a testbed for integrating deceptive geometries in architecture.

The overall hope is to contribute to a raised awareness of the importance of geometry to how we understand space, and how easily our perception can be manipulated. Because not everything is as it seems.

Keywords: Visual Illusions, Impossible Objects, Architectural Representation, Descriptive Geometry

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STUDENT BACKGROUND

ACADEMIC

19-22	Architecture and Urban Design, M.Sc. Chalmers University of Technology
18-21	Structural Engineering and Building Technology, M.Sc. Chalmers University of Technology
14-17	Architecture and Engineering, B.Sc. Chalmers University of Technology

PROFESSIONAL

21-	Graduate Structural Engineer VBK Konsulterande Ingenjörer, Gothenburg
17-18	Industrial Placement (Structures) Burohappold Engineering, London

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INTRODUCTION

OVERVIEW

This thesis is an investigation of how we understand geometry and space, and how the perception can be deceived. The exploration touches on a multitude of different subjects, ranging from visual illusions and impossible objects to depth perception, projections, reflections, hovering architecture and tensegrity structures. Although these subjects at first glance might seem somewhat unconnected, they are all in one way or another related to the concept of *deceptive geometries*. In addition to defining what characterizes such geometries, the thesis includes the development of a representation method for capturing the *perceived* geometry from certain vantage points of particular interest, as well as a speculative design proposal where some of the explored principles are tried out.

STRUCTURE

This booklet is divided into five distinct sections, this preface aside. The first gives a background and context, and serves as an introduction to the subject and the architectural discourse. Next comes an exploration consisting of a number of smaller case studies, all with the common goal to define by example what deceptive geometries can be. After that, the development of a descriptive geometry representation tool for the capturing of progression through space is presented. Subsequently, a speculative design proposal for a building by the Three-Country Cairn is given, where knowledge gained from the case studies is synthesized. In the final section the findings are discussed and evaluated, the guiding questions answered and the thesis is concluded.

PURPOSE

The overall aim of this thesis is to emphasize the importance of geometry to our understanding of space, and to show how easily our perception can be manipulated. In particular, the goal is to define the term *deceptive geometries* through application in architectural analysis and design. The intention is not to come up with a strict formulation, but rather to define by example what such geometries could be, and to showcase how some of the concepts can be used as means of creating deceptive spatial experiences.

A subgoal is to develop a representation form that can be used to capture the changing perception of a geometry when progressing through space.

GUIDING QUESTIONS

- *How can deceptive geometries be integrated in architectural design, and for what applications?*

- *How can the relation between subject and object (human and space) be captured by means of descriptive geometry, and can the representation form itself inform the design?*

These questions have been at the core of the whole process. Both are rather broad and have no simple, unambiguous answers. Nevertheless, the questions are discussed and answers are suggested in the final section of the booklet, based on the findings of the exploration, along with some closing remarks.

METHOD

A rather large part of the thesis work has been dedicated to exploration, where different relevant concepts have been investigated in separate - but related - case studies. It has been a curiosity driven process, where one discovery has lead on to a new subject, and so on. For instance, the importance of the vantage point became apparent when studying impossible objects, which lead to the realization of the need for a new representation method. The knowledge harnessed has then been synthesized and applied in a design proposal.

A combination of analogue and digital tools have been used. Physical models are contrasted by parametric models and animations, emphasizing the importance of representations.

DELIMITATIONS

The scope of the subject has a potential of being very broad with many related subjects, of which many themselves are extensive. It is no way the intention of the author to cover the full range, nor to go in depth. On contrary, the aim has been to briefly touch on a large number of aspects of the field, in order to give a comprehensive understanding. Thus, the case studies have been chosen by way of example, and not by limitation.

Concerning the design proposal, it is a speculative design which is in no way intended to be built. Therefore, little account has been taken to matters such as accessibility, buildability or ethic considerations about the chosen site. The focus is put on the geometry and the perception of the designed space.

DESCRIPTIVE GEOMETRY

Descriptive geometry can refer either to a graphical method of solving three-dimensional geometry problems, or to the theory of engineering drawing based on orthographic projections (Slaby, 1956). Henceforth, emphasis lies on the latter definition. Orthographic projections are a special case of parallel projections where the projection planes and lines are orthogonal to each other. Plan and façade drawings are examples of such projections, which describe a building geometry in a compact manner from which the metrics can easily be deducted. Parallel projections do however differ from the way we perceive the world, which is through perspective projections. Thus, these two types of projections serve different purposes in the representation of a geometry.

DECEPTIVE GEOMETRIES

Deceptive geometries is an umbrella term coined by the author. It is defined as "*any geometries that in some way feel unintuitive, get you to question their nature, or deceive your perception of reality*". Thus, it does not solely refer to impossible objects nor geometries that cause visual illusions, but does also include for instance the geometries of structures that are hard to understand due to unintuitive force flows.

Being fairly vaguely formulated, the readers have the opportunity to make their own interpretations of what deceptive geometries could be. However, the hope is that this thesis will give a good notion of the intended meaning, and inspire to the use of the term in other contexts as well.

BACKGROUND

FALSE PERSPECTIVE

With his drawing *Satire on False Perspective*, William Hogarth wanted to emphasize the importance of understanding the principles of perspective in order to create realistic representations of reality. At a first glance, everything might seem to be in order, but as soon as you look closer, you realize that a lot of what is drawn would in fact be impossible in a three-dimensional world. The drawing highlights the importance of depth cues to our understanding of our surroundings - such as perspective, shading and occlusion, as well as the impact of our knowledge and experience of the usual relative scale and behaviour of certain objects. The caption reads: "Whoever makes a design without the knowledge of perspective will be liable to such absurdities as are shewn in this frontispiece".

FORCED PERSPECTIVE

Francesco Borromini's courtyard gallery at *Palazzo Spada* in Rome is a renowned example of baroque architecture with the purpose to deceive the eye. Due to its trapezoidal shape both in plan and section, with correlating reduced column sizes, a forced perspective is created that - from the vantage point outside the corridor - gives the illusion of the colonnade to be more than twice as long as it actual is (Roker, 2014). The statue appear to human sized but is in fact only sixty centimeters tall.

In a similar fashion, the *Piazza Retta* in front of St. Peter's Church in Rome was designed by Gian Lorenzo Bernini in 1657 to enhance the perceived height and grandness of the church's façade (Fazio, Moffett & Wodehouse, 2013).

Figure 1: Satire on False Perspective by William Hogarth (1754).

Figure 2: Francesco Borromini's Gallery at Palazzo Spada, Rome (1635).

IMPOSSIBLE OBJECTS

M. C. Escher is a world renowned graphic artist, who made numerous incredible woodcuts, lithographs and drawings during the 20th century. He is famous for his mathematically inspired motives, not the least for integrating impossible objects - i.e. shapes that would be impossible in reality - in his works. *Ascending and Descending* from 1960 is one such example, where a neverending, continuous staircase is depicted. He also made intricate dynamic geometric patterns and tessellations, such as in his works *Day and Night* and *Metamorphosis II*, where suddenly the in-between voids turn into matter and vice versa (M.C. Escher Foundation, 2022). Escher's illustrations were in fact the inspiration and starting point for this whole thesis.

ORTHOGONALITY

A vast majority of the modern rooms are designed in an orthogonal fashion - meaning that they have vertical walls and horizontal floors and ceilings. This is the most practical use of space, as we humans stand upright as a result of the gravity acting perpendicular to the surface of the Earth. Therefore, we are very accustomed to orthogonal spaces, so non-orthogonal spaces are generally harder to readily grasp. *House X* by Eisenman Architects, is a speculative design which serves as a brilliant example of how a geometry can be deliberately designed to alter your perception of it is as you move about it. The whole model is skewed, which is obvious from most directions. However, from a certain direction, it appear to be completely orthogonal (Eisenman Architects, 1975).

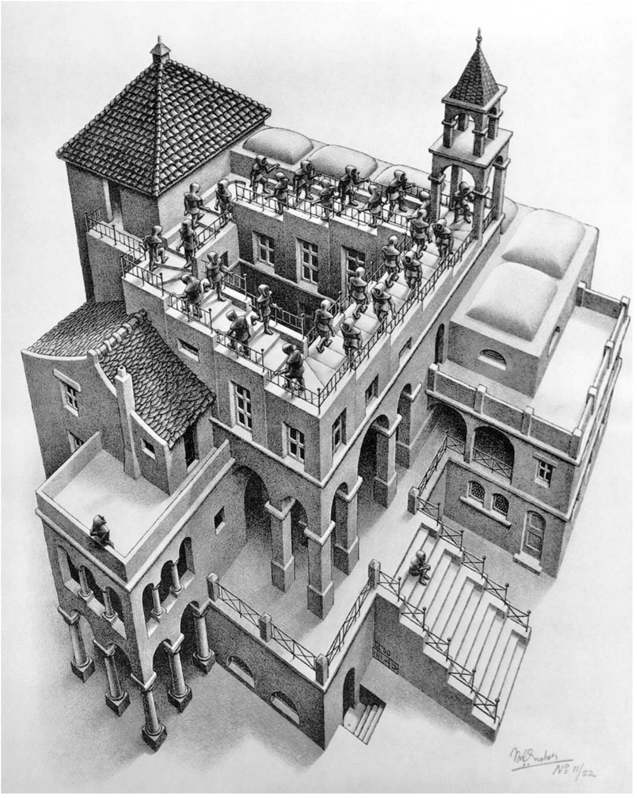


Figure 3: Ascending and Descending by M. C. Escher (1960).

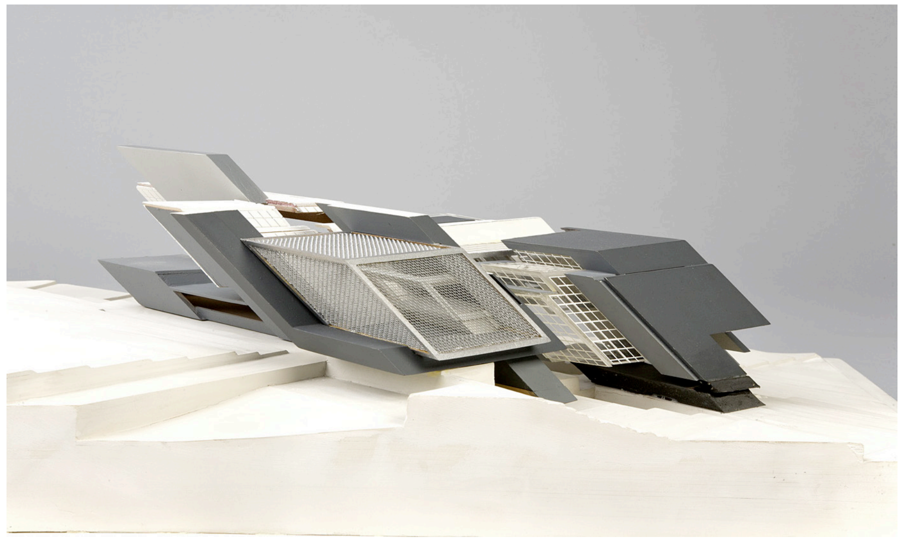


Figure 4: Model of House X by Eisenman Architects (1975).

HOVERING ARCHITECTURE

The concept of *hovering architecture* is to make a structure appear to be hovering above the ground. This effect can be accomplished in different manners, for instance by using large overhangs with hidden consoles. Another way is to use reflections, which can be achieved either using water or mirrors (Nikolić, Nikolić, Mitković, Živković & Tamburić, 2021).

An example of the latter is the *Mima Light*, designed by Mima Housing back in 2015, and which "is meant to be a house and an art work at the same time" (Sousa & Brandão, 2015). By having clad the lower section of the façade in mirrors, a visual illusion is created, making the remaining part of the building appear to be floating in space.

TENSEGRITY STRUCTURES

Buckminster Fuller was a master of *tensegrity* - a term now well established that was originally coined by him. A tensegrity structure is composed of isolated compression struts balanced by a system of tension ties, such that equilibrium is reached. A famous example of such is his geodesic dome, constructed for the Montreal Expo '67 (Fazio, Moffett & Wodehouse, 2013). A more recent example is the *Kurilpa Bridge* in Brisbane, which is a tensegrity cable stay bridge. The seemingly disordered set of struts and ties is based on complex geometric analyses and provide stability for the 120 meter long span (Arup, 2022). Common to many tensegrity structures is their intricate or unintuitive force flows, making them hard to read.

Figure 5: Mima Light in Âncora by Mário Sousa & Marta Brandão (2015).

Figure 6: Kurilpa Bridge in Brisbane by Arup (2009).

EXPLORATIONS

VISUAL ILLUSIONS

Matthew Luckiesh begins his classic book *Visual Illusions* with the statement "Seeing is deceiving" (Luckiesh, 1922, p.1). As our understanding of our surroundings to a large extent is based on interpretation, it can also be easily manipulated - a fact that is deliberately used in the creation of visual illusions.

There are numerous different types of visual illusions, and different ways of classifying them. One interesting subject for architectural purposes is *colour illusions*, some of which really highlight the importance of colour to the spatial perception (Swirnoff, 1989). A set of striking colour illusions are presented in Josef Albers' *Interaction of Color* (Albers, 1963). In contrast, the following examples belong to the category of *geometric illusions*, meaning that the deception of those is caused purely by our interpretation of the geometry.

Figure 7 illustrates the *Ebbinghaus Illusion*, which can be classified as an *illusion of contrast*. The two central circles have the exact same size, although to most people, the right one appear to be considerably larger than the left one. The illusion is a result of the sizes of their surrounding neighboring circles, as their relative sizes affect the perceived sizes.

Figure 8 is an illustration of the *Sander Illusion*. It consists of two connected parallelogram of different sizes, which in turn are bisected by one diagonal each. The diagonal *AB* appears to be significantly longer than the diagonal *BC*, although both having the exact same length. This can be classified as an *illusion of perspective*, as the hypothesis is that the tilt of the parallelogram induce a sense of depth, which affect the perceived length of the diagonals (Luckiesh, 1922).

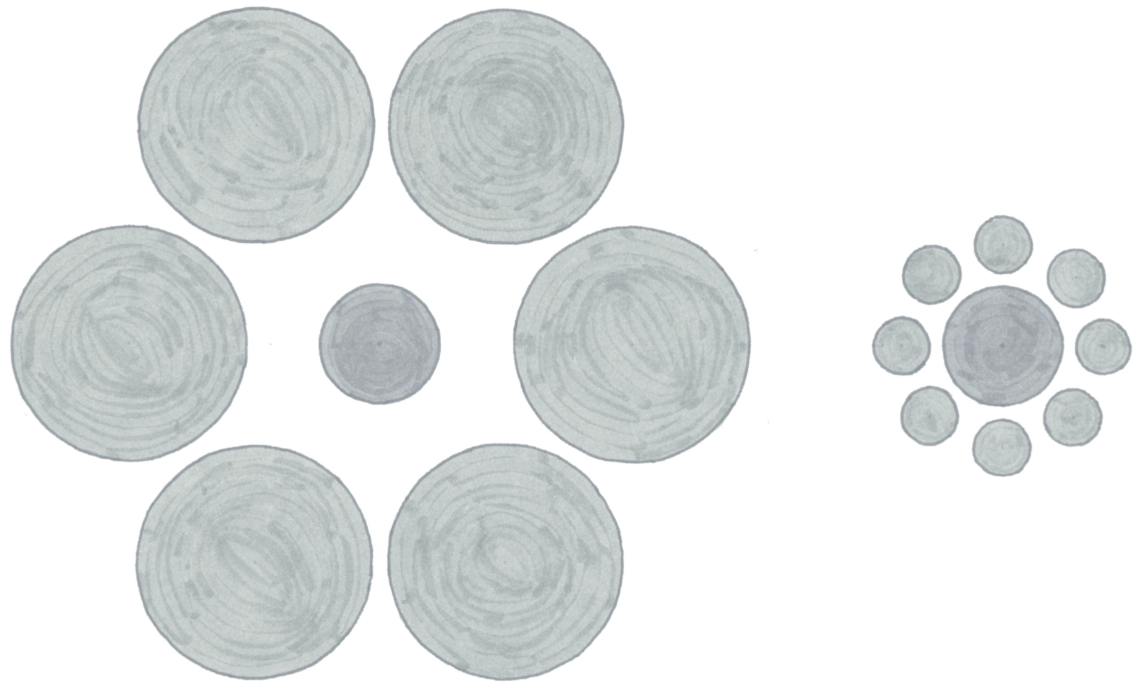


Figure 7: The Ebbinghaus Illusion - which of the two darker circles is the larger?

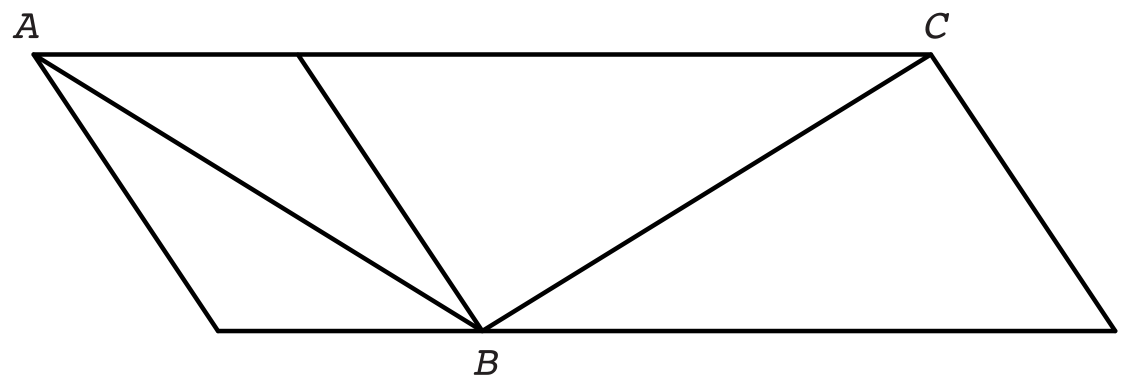


Figure 8: The Sander Illusion - which of the diagonals AB and BC is the longer?

DEPTH PERCEPTION

There are multiple different depth cues that we subconsciously utilize when we interpret our physical surroundings. In addition to purely experience based knowledge of the nature of certain types of objects - their usual relative sizes, movement patterns, etc. - we use these depth cues to transpose two-dimensional projections into a perception of depth. Binocular depth cues (*B*) require two eyes while monocular depth cues (*M*) require only one. Binocular depth cues are mainly relevant for judging the distance to relatively nearby objects, as the angle between the two sight lines converges to zero the further away you focus the eye. The selection and categorization of depth cues made in Figure 9 is the same as that presented by Christopher Kennard, professor of Clinical Neurology at University of Oxford, in one of his lectures (Kennard, 2016).

STEREOPSIS (*B*) - having two eyes gives us two slightly different perspectival images, which our brains put together to one three-dimensional image.

OCCLUSION (*M*) - objects that appear to occlude the objects behind them are perceived as closer.

SHADING (*M*) - shadows combined with lighting conditions provide useful depth information.

RELATIVE MOTION (*M*) - when moving in relation to a set of objects, the closer ones appear to be moving more swiftly.

PERSPECTIVE (*M*) - the rules of linear perspective contributes to our depth perception.

BLUR (*M*) - sharp objects appear closer than blurry ones.

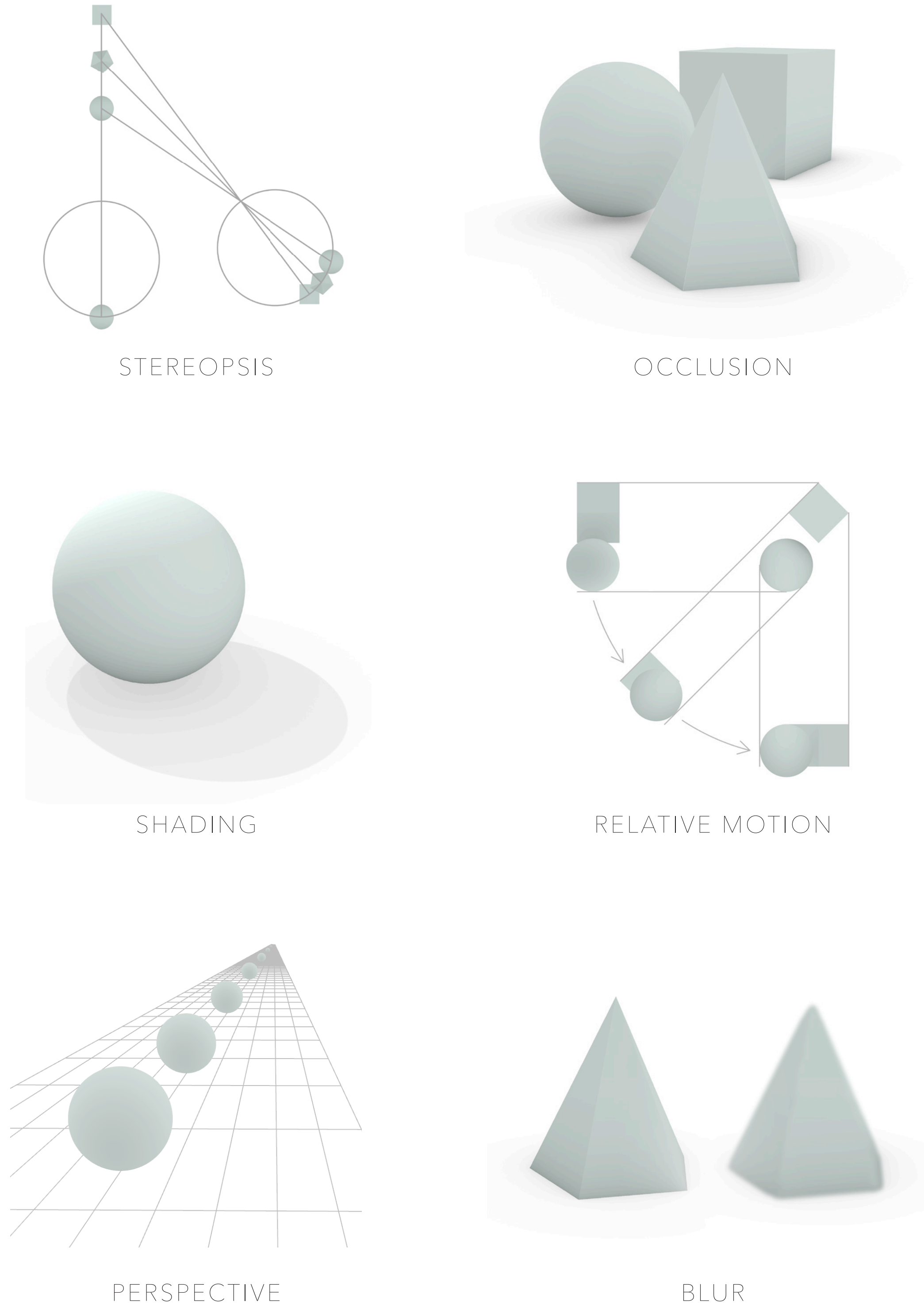


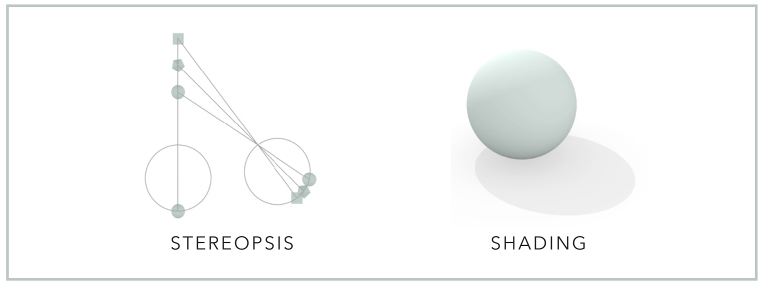
Figure 9: Depth cues crucial to our perception of depth.

PENROSE TRIANGLE

In two dimensions you can represent some three-dimensional geometries that cannot in reality exist physically in three dimensions - hence the term *impossible objects*. A master of this craft was Oscar Reutersvärd, who made numerous studies of this interesting geometric phenomenon during the 20th century. The *Penrose Triangle* was constructed as early as 1932 by Reutersvärd, but made famous by L.S. and R. Penroses in 1958, from where it got its name (Reutersvärd, 1982).

Although impossible objects cannot exist in three dimensions, it can still be possible to construct geometries such that they seem impossible from specific privileged vantage points. Figure 10 shows a cardboard model which, from a certain angle and distance, form a perfect Penrose Triangle, and an overlay of photographs of it from different rotation angles.

Due to the small scale of the model, the privileged vantage point lies rather close to the model, why the effect of *stereopsis* comes into play. Therefore, the illusion requires the observer to have one eye closed in order to achieve the two-dimensional effect shown in the photographs. The illusion can also easily be given away by *shadows* cast by the model itself. Figure 11 shows the same model in three different lighting conditions.



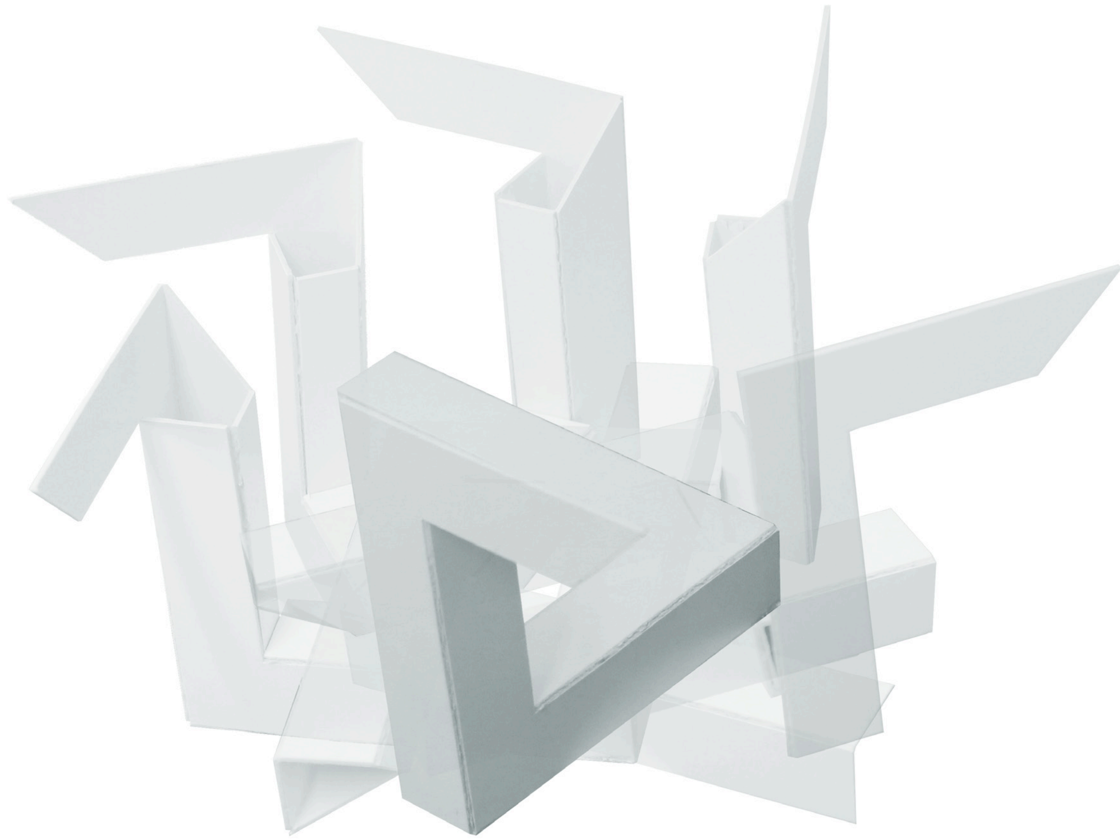


Figure 10: The Penrose Triangle - composite image of rotating physical model.

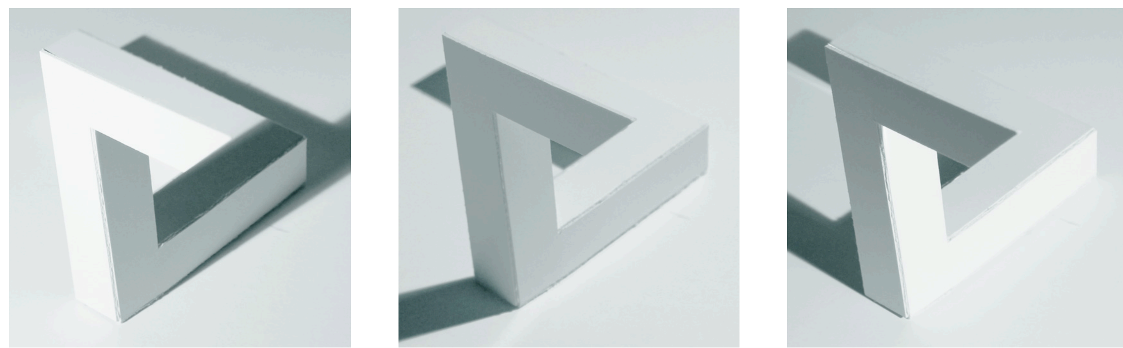


Figure 11: The Penrose Triangle - illusion dismantled by depth cues from shadows.

PENROSE STAIRS

A classic amongst impossible object and visual paradoxes is the *Penrose Stairs*, famously depicted in Escher's Ascending and Descending (see Figure 3). The staircase is drawn such that the steps seem to connect in a neverending loop, which obviously is physically impossible. Nevertheless, the illusion can be created in three dimensions, similarly to the Penrose Triangle. Figure 12 shows a wooden model, that from one point in space looks like a continuous staircase. The model is placed in the corner of two perpendicular mirrors, and the reflections reveal its actual shape and debunks the illusion.

Figure 13 shows two photos taken of the same model, but with different camera focus. This to illustrate the effect of *blur* as a depth cue, as the shift in focus indicates that there is a larger depth discrepancy between two of the steps than had they been adjacent. Similarly to the Penrose Triangle model, the small scale means that the effect of *stereopsis* is apparent.

Many impossible objects might at first glance appear logical, and it is first when you look closer you realize that something is wrong. These models can be seen as attempts of translations from three to two dimensions, while drawings of impossible objects do the opposite.

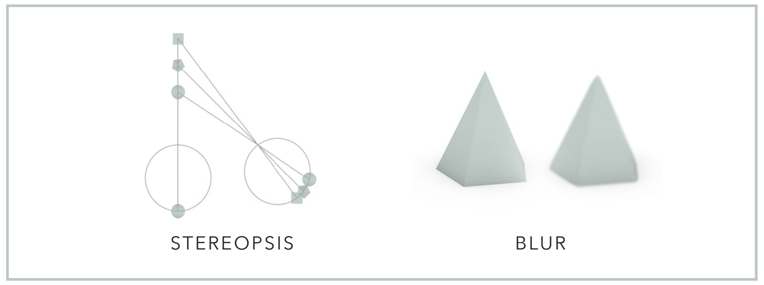


Figure 12: The Penrose Stairs - illusion deciphered by reflections.

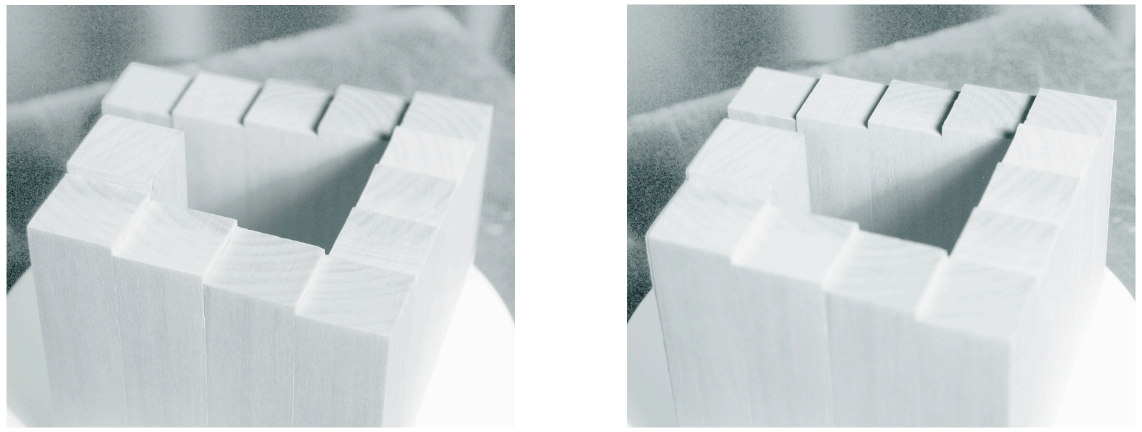


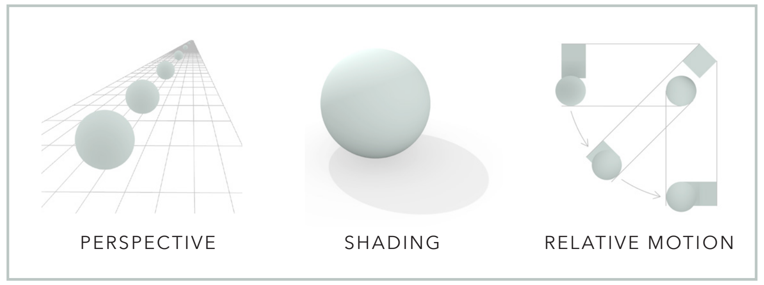
Figure 13: The Penrose Stairs - illusion debunked by depth cues from camera focus.

AMES WINDOW

One of the most compelling visual illusions is that of *Ames Window*, named after Adelbert Ames Jr., who discovered the phenomenon (Behrens, 1987). It is generated by a flat trapezoid rotating with constant speed about a vertical axis. The trapezoid is then perceived to be flipping halfway through the turn, oscillating back and forth, rather than rotating. With uniform lighting and a dark background such that no shadows give away the illusion, the two scenarios are more or less indistinguishable. The perceived *relative motion* between the rotating window and the static observer differs from actual.

Figure 14 show an Ames Window right from the front, yet it appears to be rotated due to its trapezoidal shape being misinterpreted as a rectangle rotated in relation to the observer. We are so used to orthogonal elements, so the depth perception is tricked by the apperant *linear perspective*.

Furthermore, the window itself appear to have a depth, which is induced by the darker areas that we subconsciously mistake for *shadows*. However, the trapezoid has only the thickness of a cardboard, which is strikingly revealed in Figure 15 where the model has been rotated such that either itself or its shadow becomes nothing but a thin line.



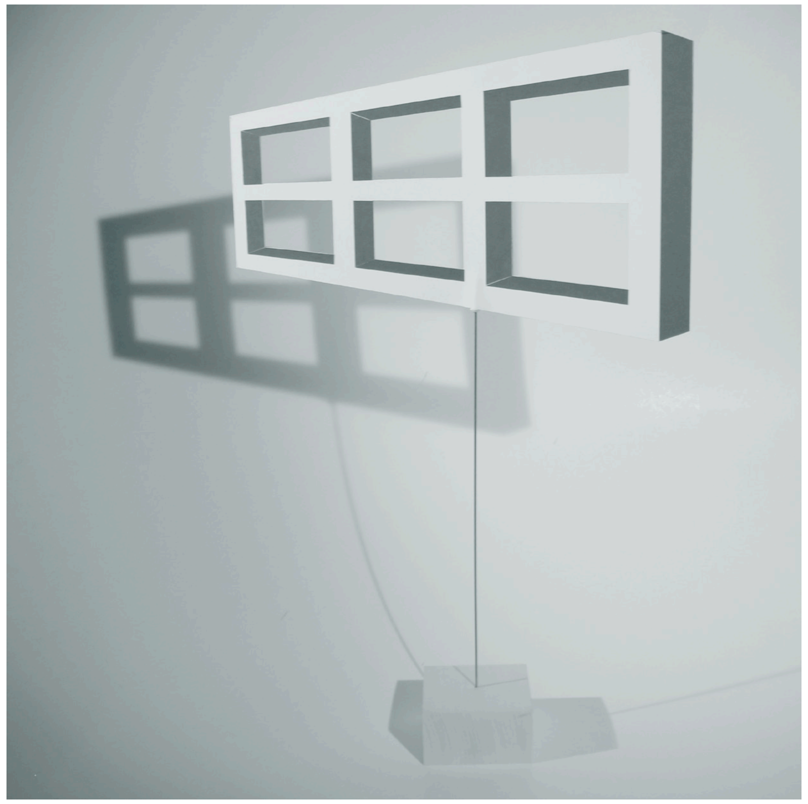


Figure 14: Ames Window - illusion of having depth, and flipping while rotating.

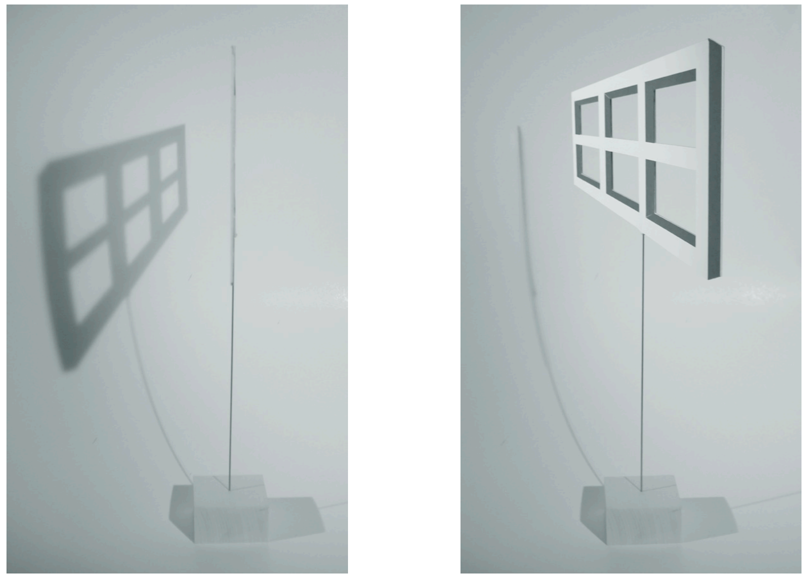


Figure 15: Ames Window - depth illusion dismantled by shadows.

AMES ROOM

Another renowned illusion of perspective by Ames is the *Ames Room*. Figure 17 shows a model of an Ames Room from above and from the front, revealing its peculiar geometry with sloping floor and ceiling, and only two parallel walls. Nevertheless, from one privileged vantage point, the room looks just like any other orthogonal room. This is an example of *forced perspective*, where the whole shape of the room is based on a specific vantage point and the shape of the room it is meant to mimic. The concept of forced perspective is further explained on subsequent pages.

The illusion becomes apparent when you introduce objects (or persons) of relatable sizes into the space. Figure 16 shows a model photo where two scale figures have been placed in in the room. The woman to the right clearly looks taller than the man to the left, but in reality they have the exact same height. The apparent size difference is merely a result of the left figure being placed much farther away from the camera - it is the distorted proportions of the room that gives the illusion of the two corners being equally close, as we subconsciously interpret the space as orthogonal. The photo has been taken with a high aperture setting in order to get both figures equally in focus, as the *blur* would otherwise give the illusion it away.

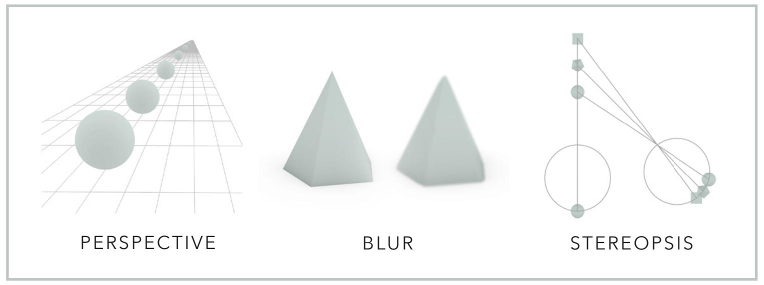


Figure 16: Ames Room - which of the figures is the taller?

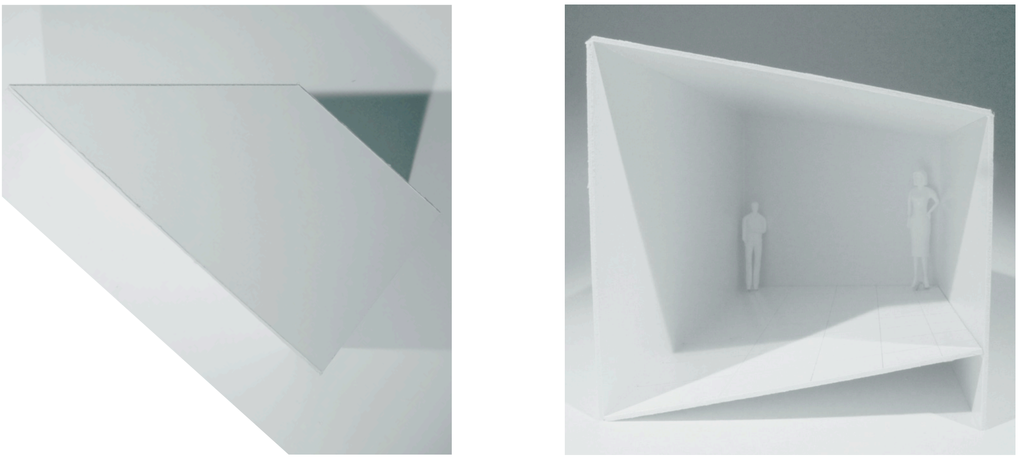


Figure 17: Ames Room - illusion debunked by understanding the shape.

FORCED PERSPECTIVE

The apparent size of an object from a specific vantage point is measured by the number of degrees (radians) it fills up in our field of view. The isosceles triangle created by the tangent lines can be split up in two right angle triangles, from where basic trigonometry can be used to determine the apparent size according to the expression for θ_{exact} below, given the width (or height) w of the object and the distance *d* to it from our vantage point pt. For distances greater than two times the size of the object, the size can be accurately approximated by the simpler expression for θ_{approx} (Hess, 2017).

$$\theta_{exact} = 2\ tan^{-1}\frac{w}{2d}$$

$$\theta_{approx} = \frac{w}{d}$$

Thus, by using this formula, you can make objects of different size appear equally large by placing them on certain correlating distances. In Figure 18, the blue rectangle represents a projection of the black rectangle, where the two appear to be of equal size from the vantage point *pt*, although *w' << w*.

In addition to the examples presented in the Introduction and on the previous page, this principle has also historically been used in stage design. This is an ideal application of forced perspective, as you have a limited space but want to create a feeling of grandness, and the audience is seated in a dedicated area. Figure 19 is a generic illustrative plan inspired by the stage designs of Ferdinando Galli Bibiena from the early 18th century (Pagliano, 2016), showcasing the principle.

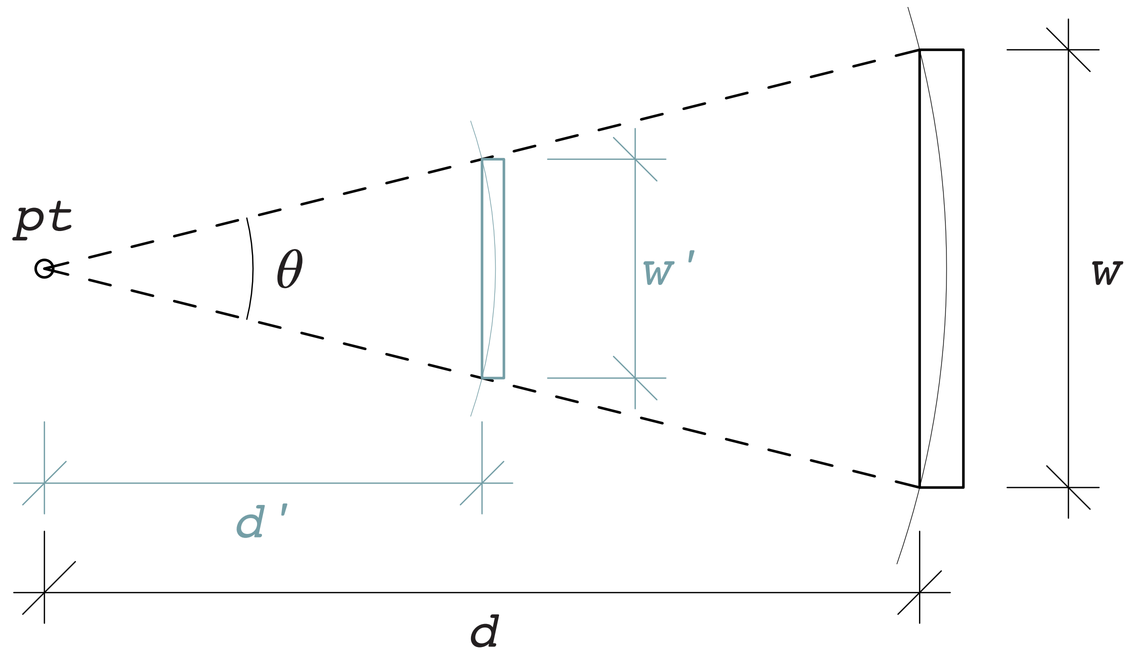


Figure 18: The principle of forced perspective.

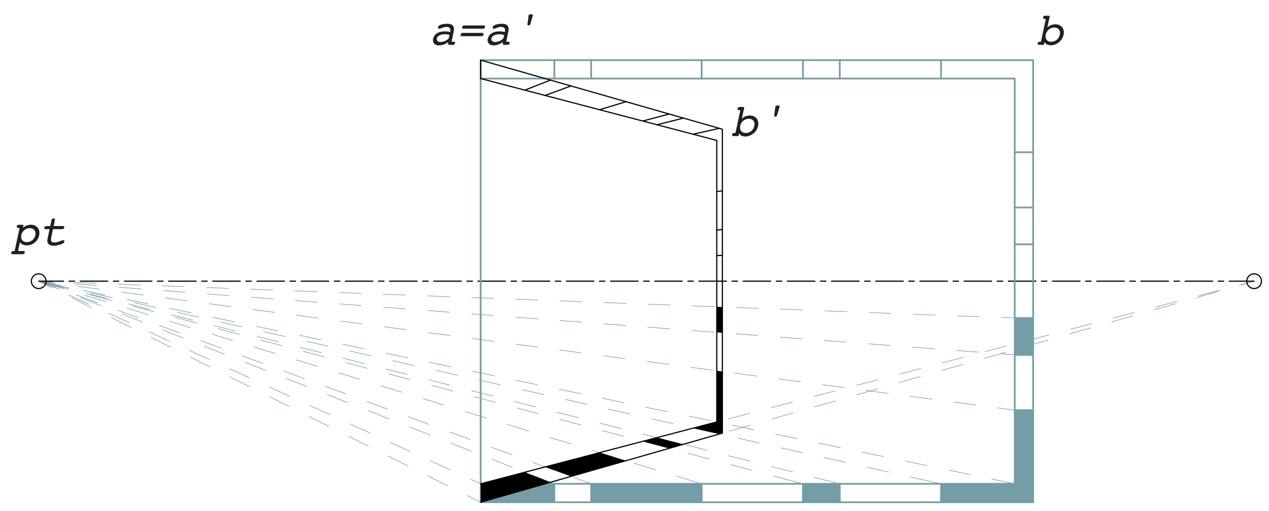


Figure 19: Construction of stage design plan using forced perspective.

PROJECTIONS

Our spatial understanding of our surroundings is based on projections of the three-dimensional world onto our retinas, which our brains then combine and reinterpret as something three-dimensional. This process is not so unlike that of creating architectural drawings as representations for buildings, where two-dimensional projections describe three-dimensional geometries. However, there are different types of projections. We experience the world through *perspective projections*, as all projection lines end up in the same focal points (in the eyes). In contrast, the conventional architectural drawings - plans, façades and sections - are *parallel projections*, where instead all projection lines are parallel to each other. This is also known as *descriptive geometry* (and more specifically *frontality*, when referring to the projections oriented parallel to the geometry planes they represent). It enables metrics and proportions to be easily deducted, and is indeed a very efficient way to describe a geometry. Nevertheless, it differs from how we experience the geometry in reality.

Figure 20 illustrates an example of *anamorphosis* (or *anamorphic projection*) of a pattern projected onto a curved wall. The correct motive is only seen from the privileged vantage point from which the projection has been performed.

Figure 21 shows a three-dimensional *ambigram*, which creates three distinctly different geometric shapes when projected along the three axes (an equilateral triangle, a square and a circle), along with parallel projections in three intermediate directions. Although a different motive, the inspiration for this figure came from the book cover of Gödel, Escher, Bach by Douglas Hofstadter (Hofstadter, 1979).

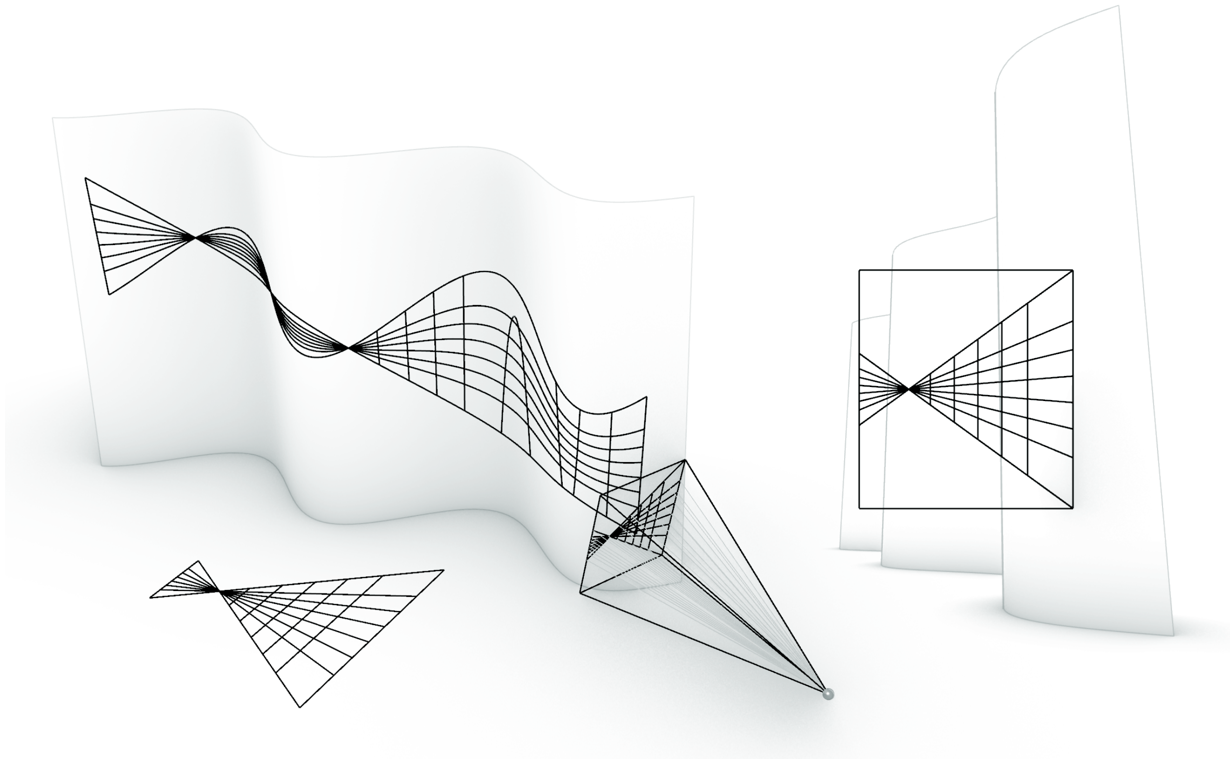


Figure 20: Anamorphic projection of geometry on curved wall.

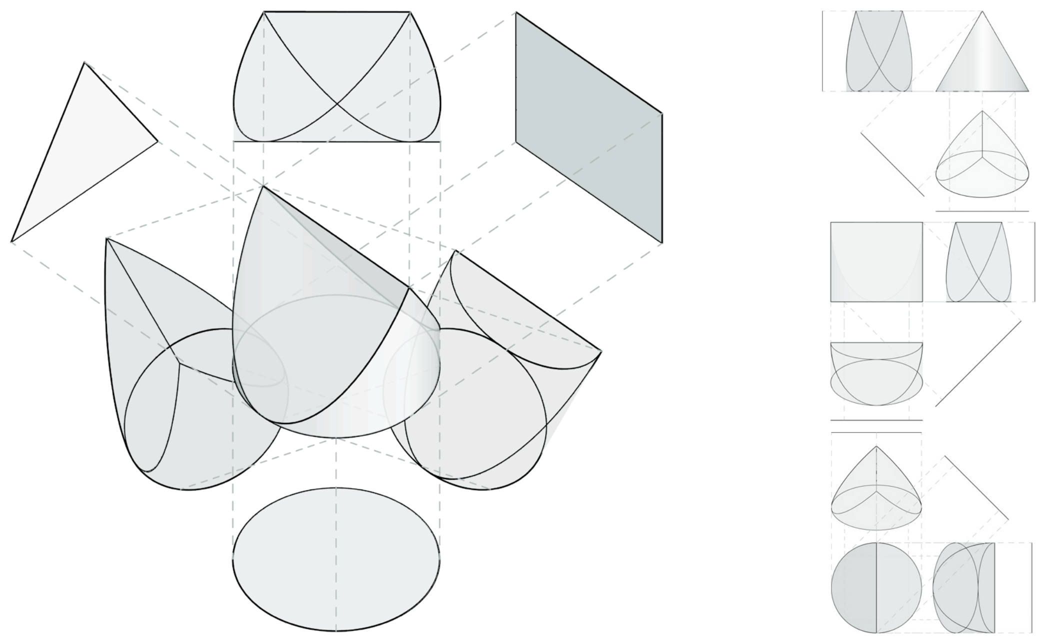


Figure 21: Parallel projections of three-dimensional ambigram.

REFLECTIONS

Mirrors and reflections can be used both for creating illusions and deciphering them. The latter is exemplified in Figure 12 where the illusion of the Penrose Stair is debunked when the actual shape of the geometry is exposed from other angles. Because with reflections, perspective projections of multiple vantage points can be obtained from one and the same vantage point, as the direction of the projection lines is changed.

In Figure 22, there appear to be a circular mirror facing the observer, yet reflecting a model placed behind it. However, when looking closer, the illusion is given away by the shadows cast by the mirror - it is in fact not circular, but elliptic. This is an example of an anamorphic projections, as an ellipse seen from a certain vantage point form a perfect circle. Similarly, a circle seen from any other direction than from straight on creates an ellipse. The inspiration for this experiment was an illusion created by the artist Olafur Eliasson, where he used a large elliptic mirror to deceive the viewers (Eliasson, 2019).

Figure 23 shows commonplace objects, placed in the intersection between two perpendicular mirrors. The reflections and the actual objects merge together, and the border between reality and perception gets blurred.

Similar effects can be obtained by the use of water mirrors, where the reflection becomes an important part of the overall perception of the geometry. *The Ambiguous Cylinder* is another interesting example of deceptive geometries achieved by reflections. Mirrors have also been frequently used by magicians throughout history, for instance with *the Sphinx illusion* (Luckiesh, 1922).

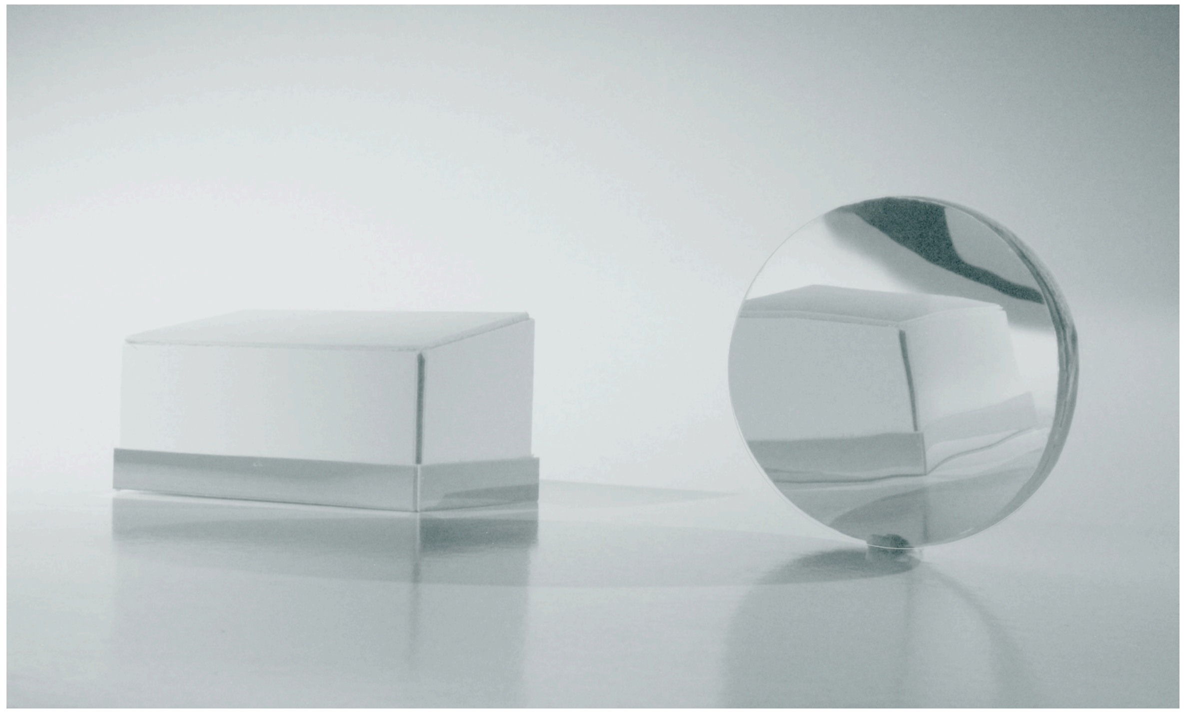


Figure 22: Mirror illusion - how can the circular mirror reflect the model behind it?

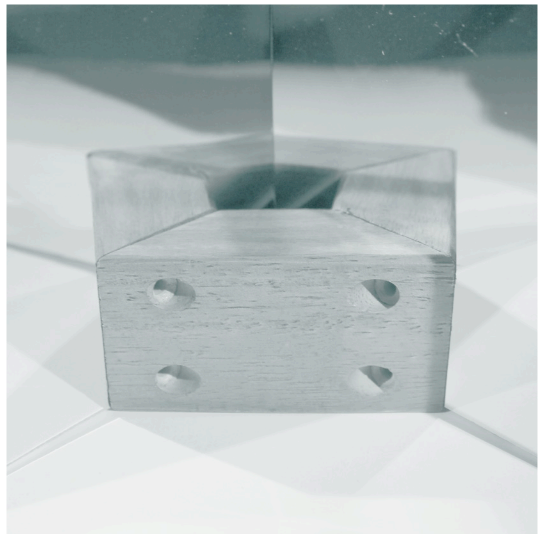


Figure 23: Illusion created using perpendicular mirrors.

DECEPTIVE FORCE FLOWS

There is an intrinsic relation between form and force; geometry and structural behaviour. This is more apparent in certain types of structures, such as tensegrity structures, where the force flows have informed the design. As it is the perception of the geometry that is important to what is to be defined as deceptive geometries or not, the cause is secondary. Thus, intricate or unintuitive force flows can result in deceptive geometries, as their structural behaviour and equilibriums are hard to understand.

Figure 24 shows an example of this, where the first impression is that it should not be possible for the structure to stay in equilibrium, as the top part is connected to the lower part only by chains (or ties). Structures like this are also known as "anti-gravity structures" for their apparent way of defying the gravity.

Figure 25 exhibits a tensegrity icosahedron from two angles. This is another classical example of tensegrity, where the struts are suspended and locked in place in a network of ties. The deceptive effect of both these examples would be enhanced by using more massive (heavy) compression and/or bending elements, and thinner ties.

Another maybe more obvious example of deceptive geometries caused by deceptive force flows is buildings with large overhangs. When the consoling parts become large enough, and the structural elements are hidden from sight, you start to question how the systems can remain stable. This typology can be seen as a subset of hovering architecture, but where the deceptive effect is achieved by the structural system rather than by mirrors and reflections.

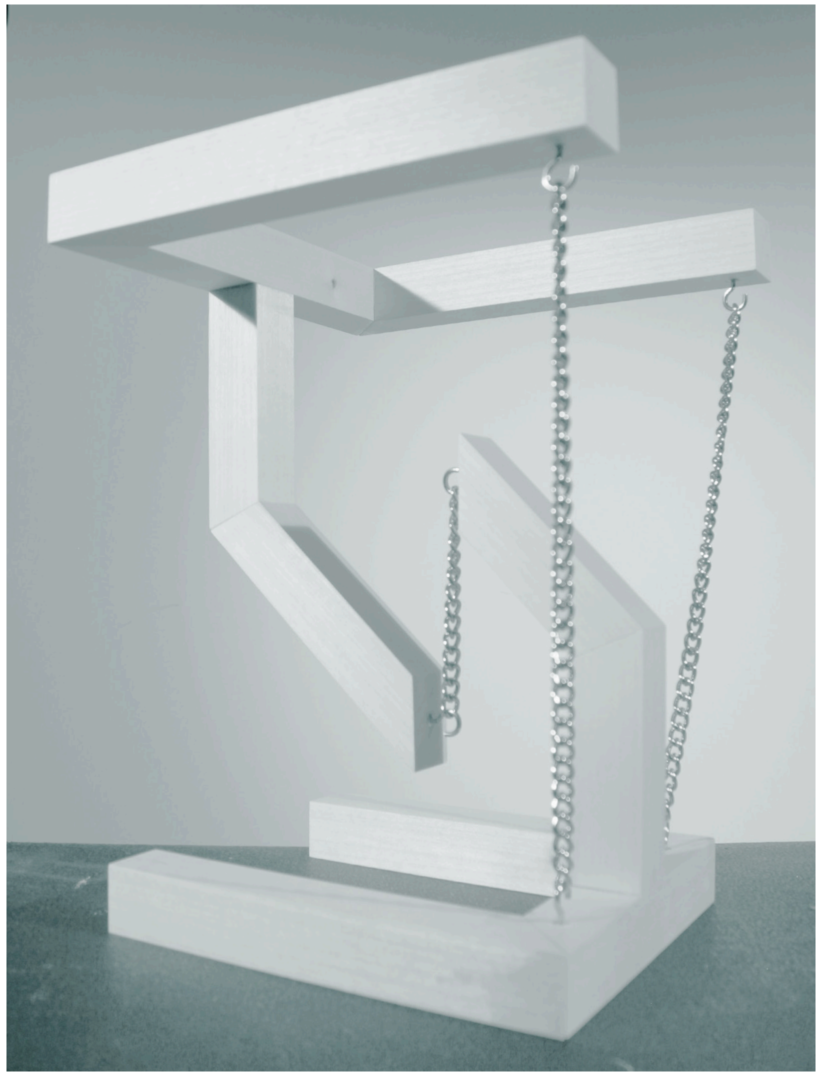


Figure 24: "Anti-gravity" structure.

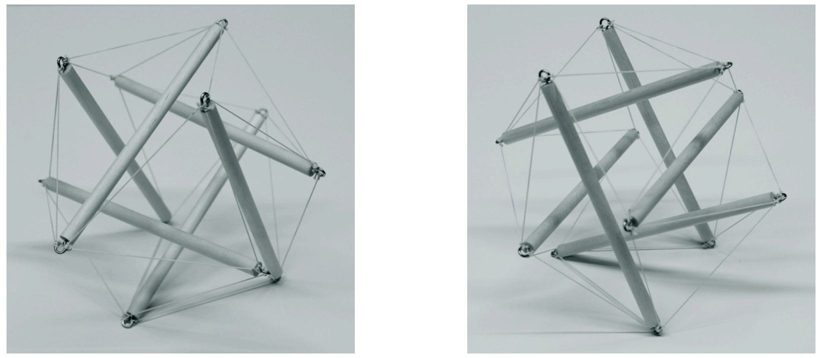


Figure 25: Tensegrity Icosahedron.

REPRESENTATION

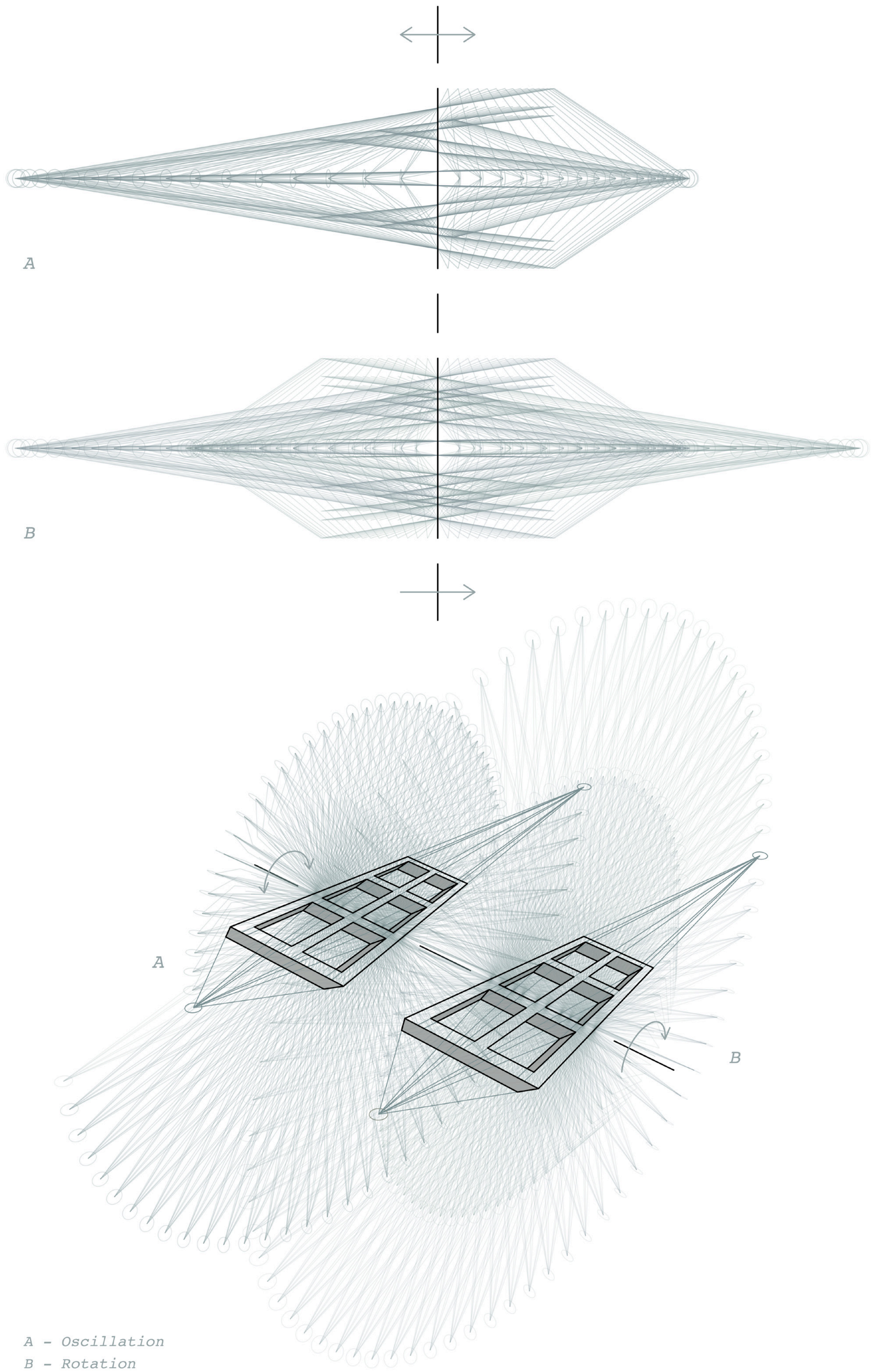
TRACING OF PERSPECTIVE LINES

The Ames Window illusion (see page 24) has served as a first case study in the development of a representation method to describe and decipher deceptive geometries. The illusion makes a rotating trapezoid appear to be oscillating, but by comparing the two behaviours, the illusion can be debunked. In Figure 26, the artificial perspective lines and vanishing points for one oscillating (A) and one rotating (B) trapezoid have been traced. Viewed from in front, the difference is hard to distinguish also in the representations - hence the illusion - but viewed from other directions, a distinction can be made between the two cases. For the rotating window (B), the two vanishing points make a full turn each, whilst for the flipping window (A), an asymmetry appear in the recorded geometry due to the vanishing points only covering half a turn each.

The illusion is as most apparent when the window is seen from in front, with a black backdrop and homogeneous lighting conditions - the two-dimensional effect created becomes more prominent the more depth cues that are eliminated. You could argue that the information loss in the transition form three to two dimensions is here utilized in creating the deception.

In the case of Ames Window, the observer is static while the object is moving, in contrast to most cases in the built environment. In the subsequent pages, the Ames Room serves as testbed, where instead the object is static while the observer is moving.

Figure 26: Freezed movement - recorded frames tracing the vanishing points of rotating and oscillating Ames Window. ▸



A PROGRESSION THROUGH SPACE

The conventional way of representing a building with orthographic projections, i.e. plan and façade drawings, is very convenient for describing its geometry in a compact manner. However, we seldom view a building from these idealized directions. Furthermore, our eyes receive perspective projections and not parallel projections. Therefore an extension to conventional descriptive geometry is developed to better capture the shifting *perceived* geometry in a progression through space. Because what characterizes deceptive geometries in three dimensions is that they are perceived very differently depending on the vantage point from which they are viewed. For architectural design in general, but for the design of deceptive spaces in particular, the changing experience of the space as you move through or about it is of outmost importance. Thus, the relation between subject and object is desirable to evaluate, both for analysis and design purposes.

Descriptive geometry and frontality is typically concerned with parallel projections only in a few (often three) directions. In contrast, the shifting experience along a continuous path can be described by infinitely many perspective projections. In Figure 27, the Ames Room is described by a multitude of perspective projections taken along a three-dimensional path, shown in perspective as well as parallel projections. The colour gradient represent the progression through time. Here, the number of projections make any individual projection near to impossible to read - however, the recorded shapes themselves tell something about the varying visual stimuli along the path. Nevertheless, finding the right balance between and a suitable discretization of the continuous flow is crucial for a meaningful deciphering of the illusion, and is further iterated below.

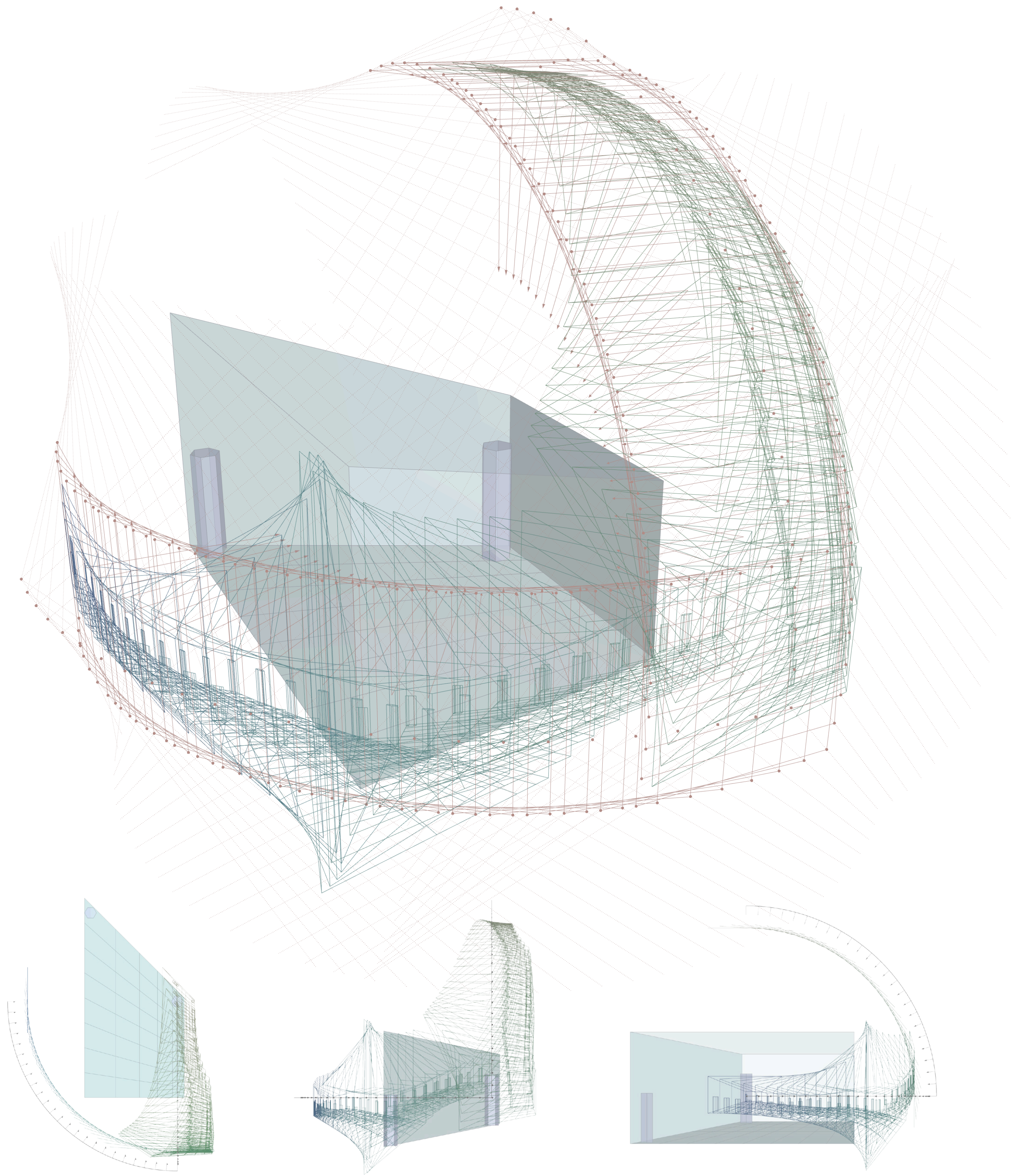


Figure 27: Iteration of finding a representation to decipher and describe the Ames Room from along a path.

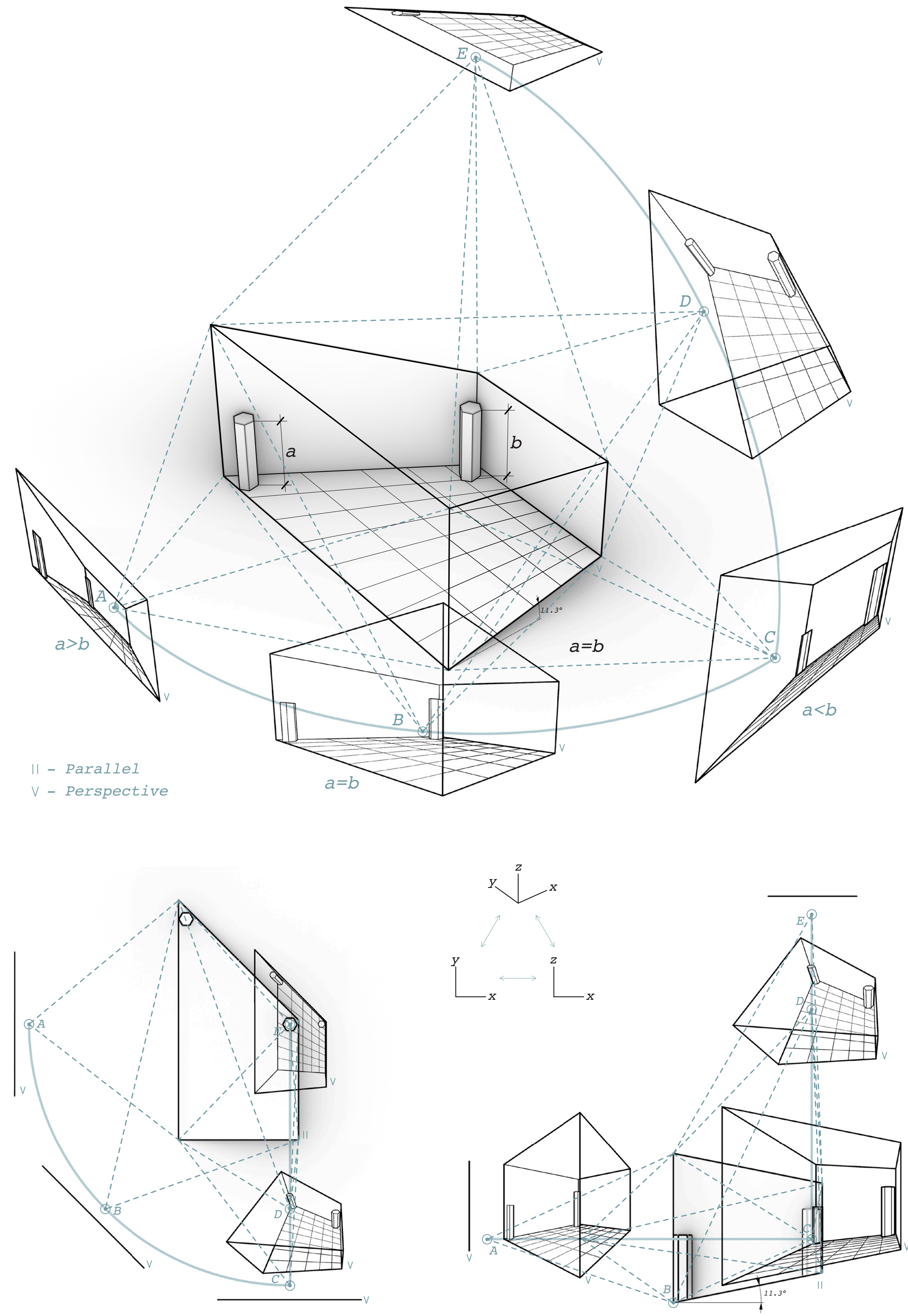
LAYERING OF INFORMATION

A complexity arises from the strive to represent multiple perspective projections in one drawing. Therefore, the line weights, line types, line colours and annotations become crucial to the separation of information and achieving a representation that is easy to read.

In the iteration shown in Figure 28, letters have substituted the colour gradient to indicate the progression through time and space, with the benefit that they are easier to refer to. The actual geometry and the projections are drawn in black for an easy comparison, whilst the path and the projection lines are blue, making this additional information layer easy to distinguish from the rest. In the case of Ames Room, one intermediate view between the frontality views is considered enough to decipher the illusion. The perceived size of the two identical columns is outlined in the three projections taken from the part of the path which is parallel with the XY plane, to show how the perceived relationship between the two is altered.

By including a plan and a front view (parallel projections), this composite drawing provides an understanding for both the actual and the perceived geometry as you move about it. By aligning the perspectives with the normals of their corresponding projection planes, the spatial feeling of movement is captured better than when aligning them with the picture plane, although with a slightly decreased readability.

Figure 28: Perspective projections of Ames Room from a selection of vantage points in a progression through space. ▸



DESIGN

SPECULATIVE DESIGN

The intention of this design proposal is to explore the concept of deceptive geometries in architecture, and to apply the developed representation method in the design of a conceptual space. Thus, the designed building is meant to serve as a proof of concept and is not in any way intended to be built. Therefore no attempts regarding feasibility have been done - otherwise important matters such as accessibility, buildability and economics are neglected in this study. The aim is to capture a progression through a deceptive space, and to represent the experience and shifting perspectives with descriptive geometry, as well as using the principles of deceptive geometries to design an interesting architectonic space that raises spatial awareness and intrigues.

THE THREE-COUNTRY CAIRN

At the northernmost tip of Sweden, the country borders of Sweden, Norway and Finland come together in a so called tripoint - the northernmost international tripoint on the globe. 11 kilometers from the nearest town, surrounded by very scenic landscape and accessible only via hiking trails, the spot is marked by the so called *Three-Country Cairn* (*Treriksröset*). It was orginally a cairn, but since 1926 the concrete monument seen in Figure 29 marks the spot (Three-Country Cairn, 2022, April 1). One could argue that this is not an optimal placement for a building, as it could reduce the genuine experience of this remote location. However, this is of no concern here as it merely serves as a speculative test bed for investigation, ethics considerations aside.

Figure 29: The Three-Country Cairn.

Figure 30: The country borders of Sweden, Finland and Norway coming together in a tripoint.

SYMBOLISM

The building is intended to be a symbolic representation of the tripoint, with three axes coming together - symbolizing the country borders - and each country getting its own room with its own entrance, and where the experience of the space is shifting as you move between the rooms. The idea is that your perception of the other rooms - or countries - should differ depending on where you stand, yet from the center of the building, all rooms should look just the same, and just as illusive. This serves as a metaphor for that we are all the same, independent of nationality; it's just the perception of the others that depend on where you stand, and that this is something that's fully mutual. In the end we are all under the same roof. You only need a change of perspective - literally speaking.

PROGRAM

In addition to a meeting place for the three neighboring countries, the building is meant to serve as a tourist attraction and a space for experience. Furthermore, the whole building is lifted from the ground on columns and shear walls, and the space created underneath function as a wind shelter, where the weary travelers can unload their backpacks. The interesting geometry and the multiple ways to move in and about it - not to mention the illusiveness - makes it the ideal playground too.

First and foremost, though, with multiple concepts of deceptive geometries integrated, it highlights the importance of geometry to how we perceive our surroundings. Rather than being an exhibition space, its an exhibition *of* space.

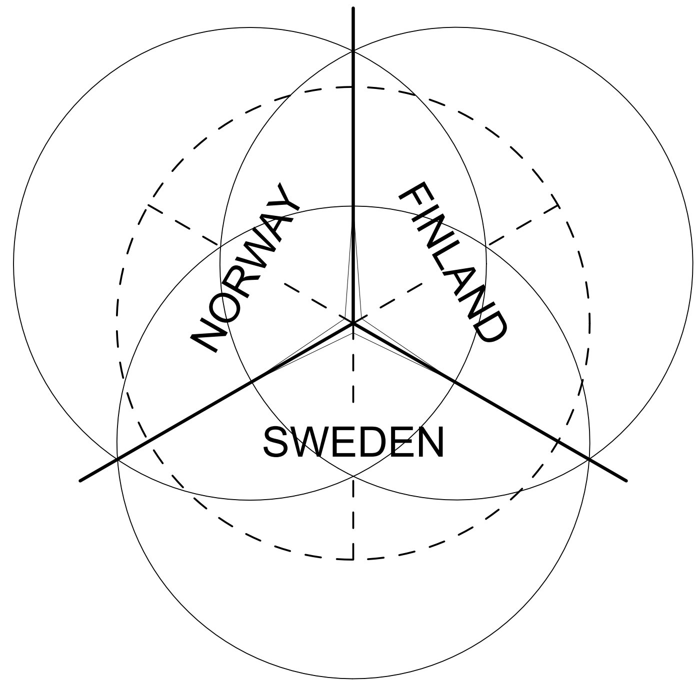


Figure 31: Schematic diagram of the tripoint.

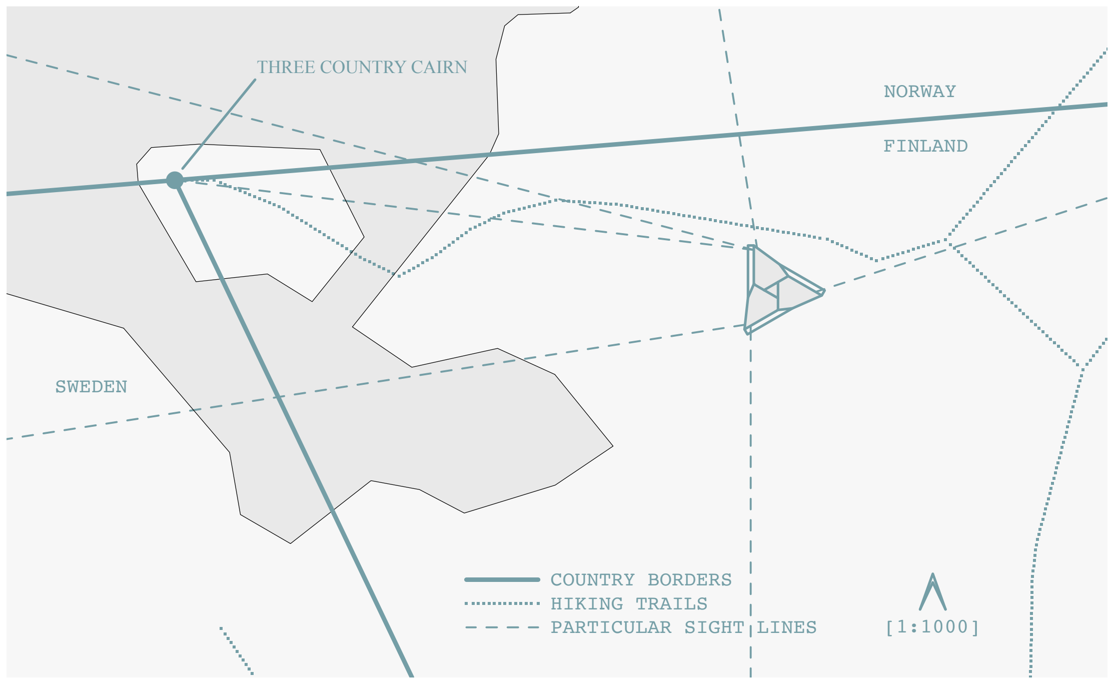


Figure 32: Site plan.

CONCEPT

The choice of site is to a large degree motivated by the symbolism of the tripoint. The geographic country borders provide the basis for a triaxial order, rather than the typical orthogonal one, which introduces a geometric complexity which is suitable for deceptive geometries and creating an unpredictable spatial experience. Although having a complete rotational symmetry, the geometry is hard to read from the ground - a fact emphasized by the building having almost no horizontal surfaces. The floor and roof shapes is achieved by flat surfaces being folded into their three-dimensional shapes - speaking of dimension transitions (see Figure 34). The only horizontal floor patch is the equilateral triangle which constitute the center of the building. Here, the country borders symbolically come together, symbolized by a glazed crack in the floor, which divides the building in three identical pieces, and through which you can get a glimpse of the space underneath the building.

The building has been placed to greet the hikers to the site, yet not to block the view out over the lake (see Figure 32). The orientation is based both on the direction of the geographic borders, and to frame outlooks of the most scenic views out over the lake and the mountains - seen from each room towards the corresponding country.

Several of the deceptive concepts explored in the case studies have been integrated in the design. Furthermore, the developed representation strategy has been used as a design tool to inform the outcome, with the intended experience along a certain path and progression through space in focus, as will be presented on the subsequent pages.

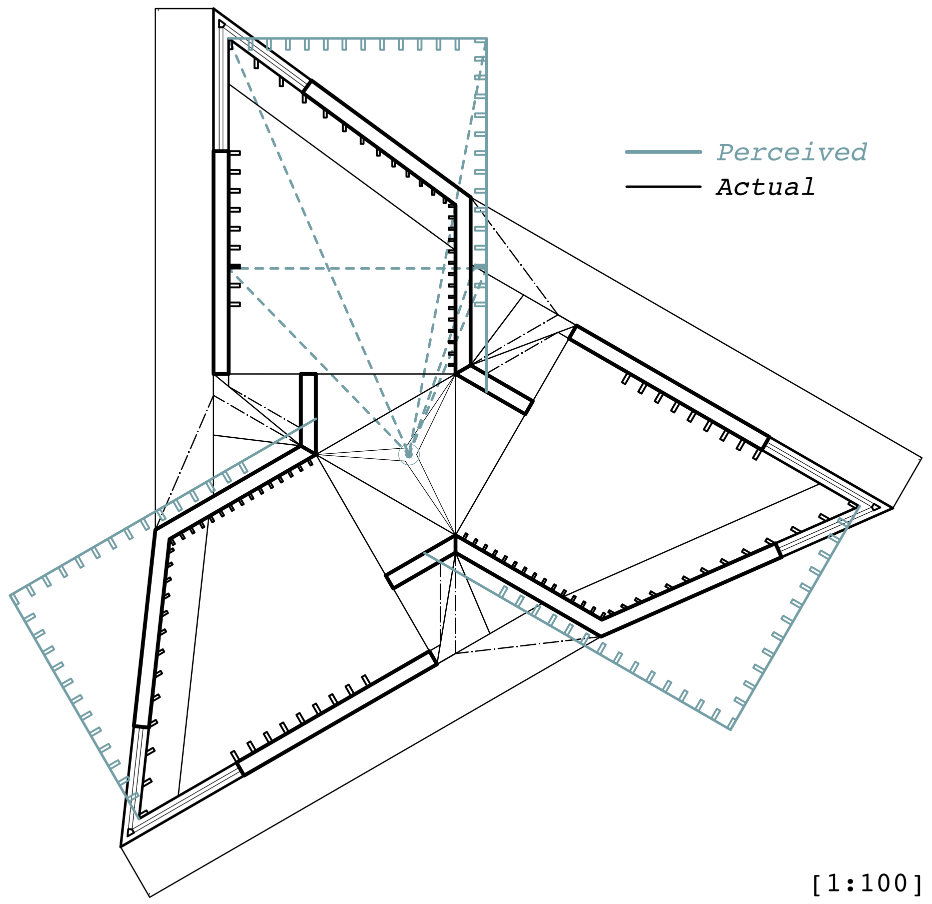


Figure 33: Plan including perceived geometry when standing in the center of the building.

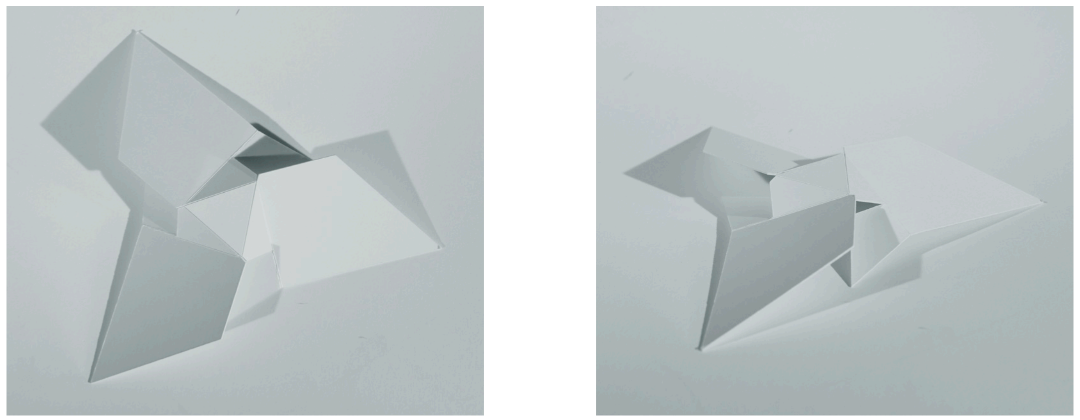
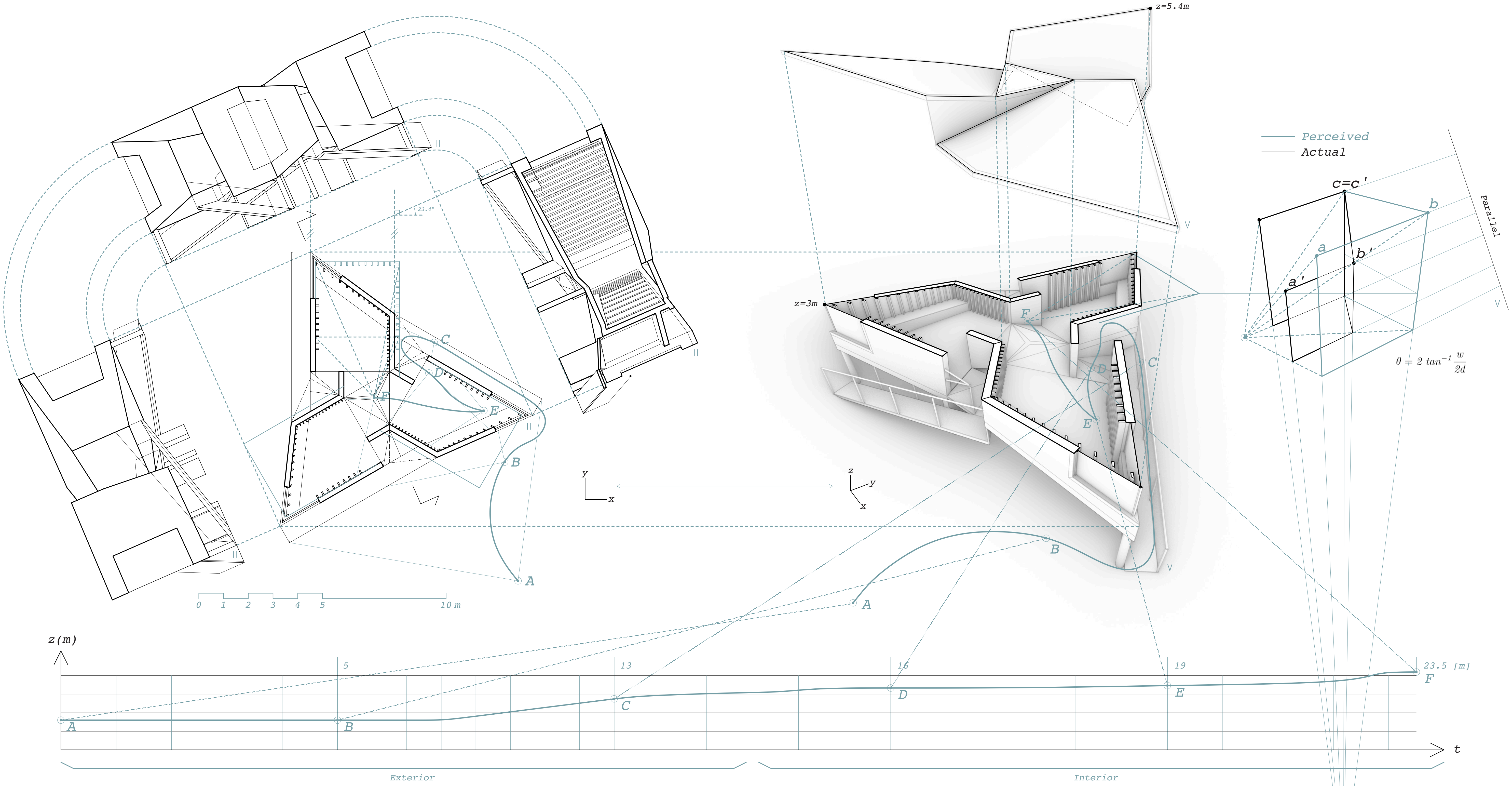
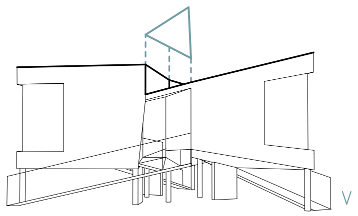
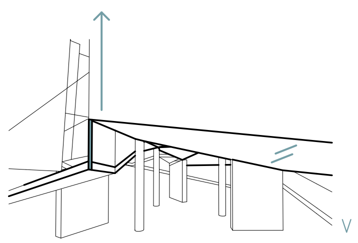


Figure 34: Physical concept model of floor shape folded from a flat piece of cardboard.

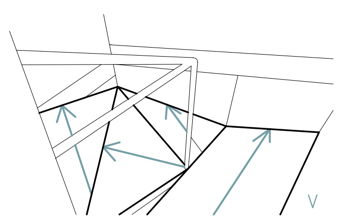




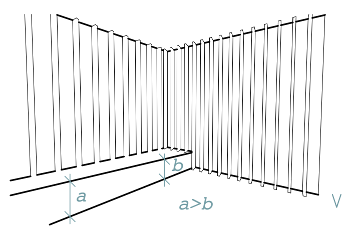
A. Triaxial Complexity



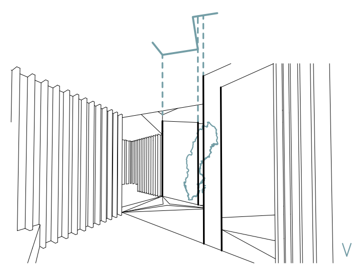
B. Hovering Architecture



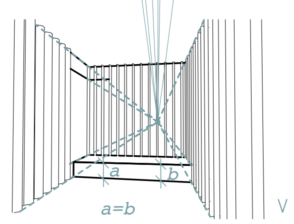
C. Non-Horizontal Floor



D. Skewed Proportions



E. Anamorphosis



F. Forced Perspective

EXTERIOR (A)

The hiking routes merge into one before the last stretch to the Three-Country Cairn (see Figure 32), meaning that most people will approach the building from the same direction, although getting glimpses of it earlier. One could imagine placing minor deceptive installations of land art along the paths, to give you a hint of what is to come, and emphasize the fact that the trekking itself is an extension of the experience. From Point A, you view the building from a distance, and the geometry is very hard to grasp and changes dramatically with the direction from which it is viewed. The triaxial symmetry and non-horizontal surfaces makes the geometry deceptive and very hard to read. Shifting directions in section can easily be confused with direction shifts in plan, and vice versa.

DECEPTIVE VOID (B)

From Point B you can appreciate the space under the building, which is deceptive itself. Firstly, the full underside of the building is clad with mirrors which creates a hovering effect of the whole building, and invites you to explore the space and see your reflections from odd angles. Secondly, the corners over the entrances to this space have no columns and seem to be completely unsupported, which gets you to question how it is possible for it to remain in equilibrium. This deceptive force flow is achieved by having the cantilevering structure hidden inside the walls, and balanced by the symmetry of the building.

Figure 35: The design proposal described with a combination of descriptive geometry and projections along a path. ◂

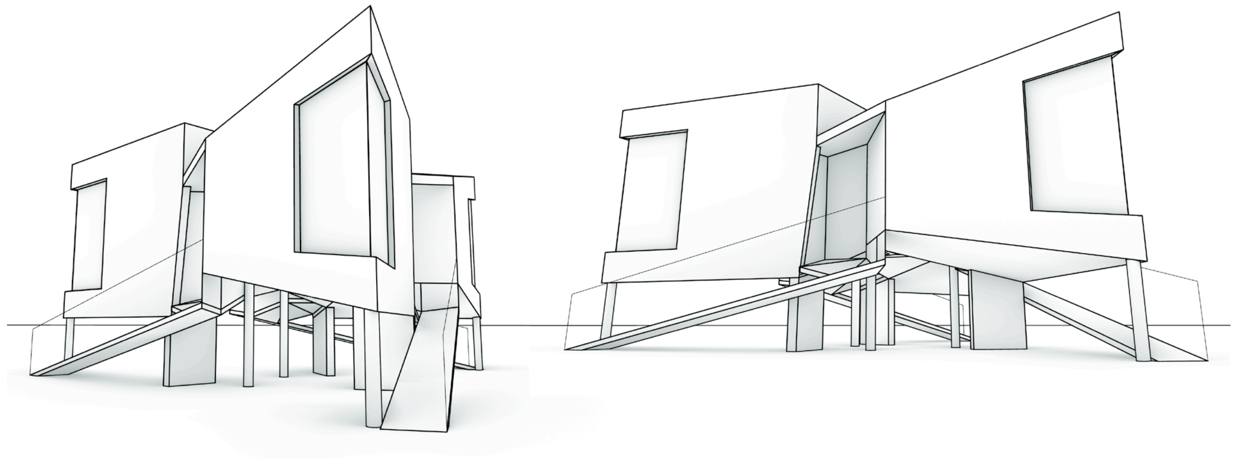


Figure 36: Abstract exterior seen from two different vantage points (the right one from Point A).

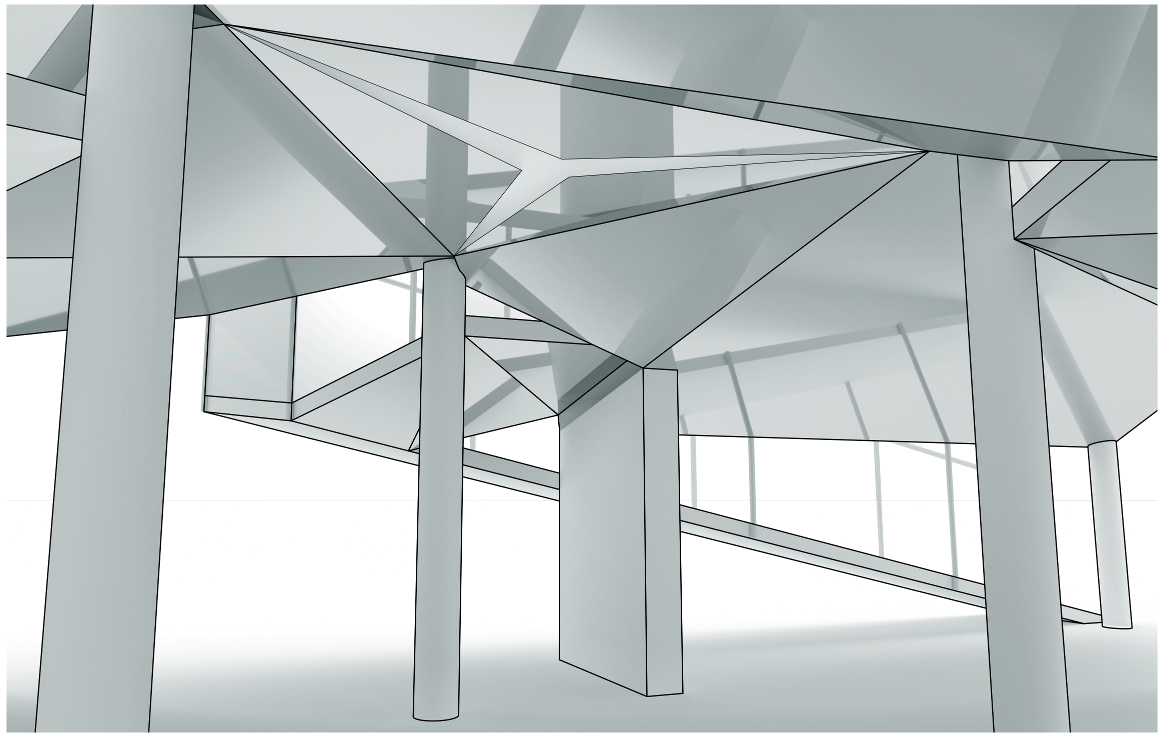


Figure 37: Deceptive space created underneath the building achieved with mirrors and large overhangs.

ENTRANCE SPACE (C)

You enter the space via one of the three ramps, which creates a long and gradual progression, almost like a hill slope to climb in order to get to the top, to reflect the landscape of the site. The narrow passage creates a directed movement, in contrast to the free movements on the outside and inside. To further build the anticipation, the path turns 180 degrees, and the enclosed entrance space offers few clues on what is to expect on the inside. At all times, you need to beware of the sloping floor, which in the same manner as stairs with different step sizes is intended to trigger you to be more aware of the surroundings. Point C shows how the ramp consists of multiple patches with different angles and slopes, seen from below in Figure 37 and from above in Figure 39.

ANAMORPHOSIS (D/E)

Well inside, you are drawn towards the corner windows of the room which you have entered as you are curious to see the elevated views from the windows. Thus, you walk over to Point E to admire the beautifully framed landscape. When you turn around, you see the proper shape of the country border of the country of the corresponding room. This anamorphic projection is only seen in its proper shape from this specific vantage point and does not make sense from anywhere else - compare views from Point D and E in Figure 38 and 39, respectively. As part of the metaphor, these projections represent that you can only fully understand a country when you are in it. However, as part of the projected image is actually located in another country's room: we are not complete without each other.

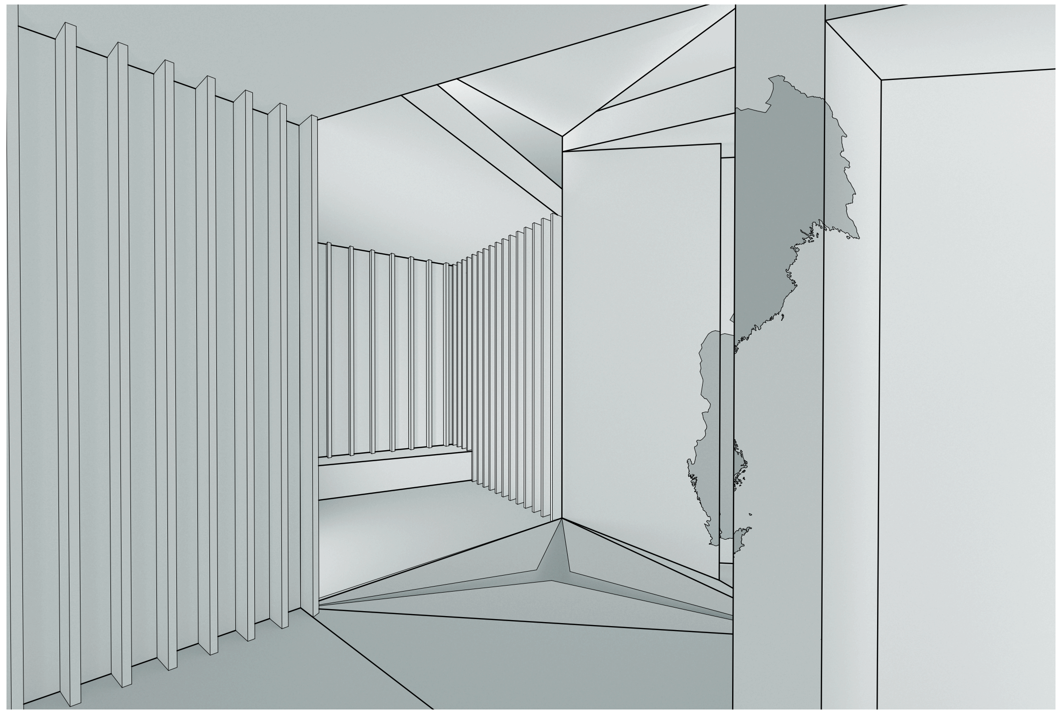


Figure 38: View towards the center of the building from Sweden's room.

Figure 39: Anamorphosis - the correct shape is only seen from one specific vantage point.

FORCED PERSPECTIVE (D/F)

As you enter the space it is clear that the room is distorted and the floor sloping, and that the bench is in fact twice as high to the left than to the right (see Figure 40). Also the columns differ in size, with much larger cross sections to the left (see plan drawing). Similar to the anamorphosis, this only makes sense from one specific point, which is in this case located at the center of the building. So after having explored the room and the views, the interesting geometry and the roof lights (see Figure 38) draws you towards the center of the space.

Here, at Point F, the orchestrated walk ends. From here all the three rooms - or countries - look the same, and the illusion of Ames Room is prominent (see Figure 41). Spotlights placed between the columns give light and shadows to enhance the illusion. The geometry of the rooms have been achieved by following the projection lines of an orthogonal room to create the forced perspective, yet to adapt the shapes to other geometric requirements from the triaxial symmetry and ramps.

The illusion represents that people from other countries may look unfamiliar and strange when seen from the outside, but as soon as you move into the country - or room - you see that this was only a result of perspective. Suitably, you now stand on the equilateral triangle which is the only horizontal floor section of the entire building - with the three country borders coming together, symbolized by a glazed crack in the floor, through which you can get a glimpse of the space underneath the building. Although being a very distinct division between the rooms, the borders are blurred as you move around, which is a celebration of the possibility of free movement between the three countries.

Figure 40: The people look normal in size, but the room is clearly distorted.

Figure 41: Seemingly orthogonal room, but the people are significantly distorted in size.

DISCUSSION

DECEPTION AS DESIGN TOOL

"How can deceptive geometries be integrated in architectural design, and for what applications?"

Deception has been used in architecture and art throughout history either to create amazement and wonder, or to emphasize certain architectonic features or elements. For instance, the case of forced perspectives have been used for both those purposes - exemplified by Ames Room and Baroque façades.

Many spatial geometric illusions work best under very controlled circumstances, both concerning the privileged vantage point and regarding lighting conditions etc. Thus, many of the case studies showcase the extremes of their respective phenomena. Therefore, their applications are quite apparent, but the specificness also makes each rather limited to one specific purpose. Also in the design proposal, the features have been emphasized in order to create a truly deceptive experience. The symbolism of the Three-Country Cairn is captured through an illusive spatial experience. However, the cases where it is desirable to create a truly deceptive experience are rather rare. Something that would be interesting to explore further is how you can introduce deceptive geometries in more subtle way in more "everyday" architecture, where the sole purpose is not to fool the observer.

Several different concepts explored in the case studies have been integrated in and shaped the outcome, and exhibits how deceptive geometries can be applied in architectural design. Independently of the actual application, the study of how to deceive the spatial perception demands an understanding of how we perceive space, which is useful for architects in general.

REPRESENTING MOVEMENT

"How can the relation between subject and object (human and space) be captured by means of descriptive geometry, and can the representation form itself inform the design?"

For architecture in general, and geometrical illusions in particular, the shifting experience of geometry as we move is of outmost interest. With the iterations done for Ames Window and Ames Room, many possible ways to capture both the actual and the perceived geometry have been tried out. The complexity lies in encapsulating all the required information in one drawing, and yet make it clear and easy to read. There are numerous variations, and it would take many iterations to find something close to an ideal combination of metrics for each individual case. Nevertheless, one set of answers is given with the representations presented in this thesis. Some metrics that have been identified as important and which could be mapped out further are: discrete or continuous, parallel or perspective, two or three dimensions, moving subject or moving object, frames parallel or perpendicular to the path.

In the design process, the representation strategy has been implemented as a design tool - both in order of thinking of a desired experience along a progression of movement, and to describe and create deceptive spaces. The former reason indicates that this method of working could be applicable not only for deceptive geometries, but for architectural design in general. By putting the experience in focus, you put emphasis on the perceived geometry, which informs the actual design. The exactness of this rather mathematical drawing approach highlights the importance of geometry to the subject.

CONCLUSION

The concept of deceptive geometries has been at the core of this thesis, as the title indicates. Being deliberately rather vaguely defined in the introduction, the hope is that the case studies have provided a good foundation for understanding what deceptive geometries can be, and that the reader is triggered to make a personal interpretation and reflect upon other examples that has not been showcased here as well. The need for the umbrella term is apparent at least in this thesis, but also enriches the narrative and the way we can talk about architecture in general.

Regarding the method of working, one can conclude that by remaining in the explorative phase for a long time, a good comprehensive foundation and understanding can be achieved. Though, the consequence is that the design application might not be as iterated as the rest. However, for the purposes of this thesis, where the application is merely meant to serve as a proof of concept, one could argue that the balance is adequate.

This exploration shows that deception can be a powerful tool in architectural design. A custom made experience can be created, and it can raise an awareness of space and the importance of geometry in the built environment. This thesis also highlight the importance of architectural representations - both in analysis and design. Many of the revelations that have come of the study are not exclusive to the subject, but useful for architectural design in general - such as the importance of the relation between subject and object. Thus, with the case studies, representation development and design application combined, the goal of this thesis can be seen as fulfilled.

CLOSING REMARKS

The striking analogy between how we visually perceive the world and the way we represent it in architectural drawings - and the importance of the relation between subject and object - was a key revelation that sprung from the explorative phase and came to affect the whole thesis. At the core of it all is the transition between different dimensions. With the way our eyes receive projections of the world, the way we represent three-dimensional objects in drawings, the way two-dimensional effects can be created in space and the manipulation of our depth perception, etc. In two dimensions you can explore a three-dimensional world that does not exist. M.C. Escher was a master of this craft, and as it was a fascination for his intricate illustrations that was the starting point for this whole thesis, it appears suitable also to include him in the closing remarks.

Application of deception and illusions in design might at first glance seem like nothing but paper architecture, especially when applied to a speculative design. However, in the strive to deceive ones perception of space, you inevitably gain an understanding of space, which is fundamental to all architectural design. The speculative aspect simply allows you to bypass restrictions and focus on the phenomena of interest.

The subject of deceptive geometries lies in the borderland between mathematics, psychology and architecture, where the perceived geometry is as important as the actual. This thesis has only started scratching on the surface. Nevertheless, the hope is that it has contributed, ever so slightly, to a raised awareness of the importance of geometry to how we understand space, and how easily our perception can be manipulated. Because not everything is as it seems.

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